



Zagazig University
Faculty of Science
Physics Department



Geometrical Optics

For
First Year Students

CONTENTS

PAGE

Chapter 1

The Nature of Light And Laws Of Geometric Optics

1

1-The Nature of Light-----	1
2- Measurements of the Speed of Light-----	3
3- The Ray Approximation in Geometric Optics-----	7
4- The Wave Under Reflection-----	9
5. The Wave Under Refraction-----	15
6. Huygens's Principle-----	26
7- Dispersion-----	30
8 - Total Internal Reflection-----	32

Chapter 2

Image Formation

37

1. Images Formed by Flat Mirrors-----	37
2. Images Formed by Spherical Mirrors-----	43
3. Images Formed by Refraction-----	58
4. Thin Lenses-----	67
5. Lens Aberrations-----	84
6. The Camera-----	87
7. The Eye-----	91
8. The Simple Magnifier-----	97
9. The Compound Microscope-----	101
10. The Telescope-----	103

Chapter 1

The Nature of Light and Laws Of Geometric Optics

1-The Nature of Light

Before the beginning of the nineteenth century, light was considered to be a stream of particles that either was emitted by the object being viewed or emanated from the eyes of the viewer. Newton came up with his model that particles were emitted from a light source and that these particles stimulated the sense of sight upon entering the eye. Using this idea, he was able to explain reflection and refraction.

Most scientists accepted Newton's particle model. During Newton's lifetime, however, another model was proposed, one that argued that light might be some sort of wave motion. In 1678, Dutch physicist and astronomer Christian Huygens showed that a wave model of light could also explain reflection and refraction.

In 1801, Thomas Young (1773–1829) provided the first clear experimental demonstration of the wave nature of light. Young showed that under appropriate conditions light rays interfere with one another. Such behavior could not be explained at that time by a particle model because there was no conceivable way in which two or more particles could come together and cancel one another. Additional developments during the nineteenth century led to the general

acceptance of the wave model of light, the most important resulting from the work of Maxwell, who in 1873 asserted that light was a form of high-frequency electromagnetic wave. Hertz provided experimental confirmation of Maxwell's theory in 1887 by producing and detecting electromagnetic waves.

Although the wave model and the classical theory of electricity and magnetism were able to explain most known properties of light, they could not explain some subsequent experiments. The most striking phenomenon is the photoelectric effect, also discovered by Hertz: when light strikes a metal surface, electrons are sometimes ejected from the surface. As one example of the difficulties that arose, experiments showed that the kinetic energy of an ejected electron is independent of the light intensity. This finding contradicted the wave model, which held that a more intense beam of light should add more energy to the electron. Einstein proposed an explanation of the photoelectric effect in 1905 using a model based on the concept of quantization developed by Max Planck (1858–1947) in 1900. The quantization model assumes the energy of a light wave is present in particles called *photons*; hence, the energy is said to be quantized.

According to Einstein's theory, the energy of a photon is proportional to the frequency of the electromagnetic wave:

$$E = hf \quad (1)$$

where the constant of proportionality $h = 6.63 \times 10^{-34} \text{ J s}$ is called *Planck's constant*. In view of these developments, light must be regarded as having a dual nature.

Light exhibits the characteristics of a wave in some situations and the characteristics of a particle in other situations. Light is light, to be sure. The question “Is light a wave or a particle?” is inappropriate, however sometimes light acts like a wave, and other times it acts like a particle

2- Measurements of the Speed of Light

Light travels at such a high speed (to three digits, $c = 3.00 \times 10^8 \text{ m/s}$) that early attempts to measure its speed were unsuccessful. Galileo attempted to measure the speed of light by positioning two observers in towers separated by approximately 10 km. Each observer carried a shuttered lantern. One observer would open his lantern first, and then the other would open his lantern at the moment he saw the light from the first lantern. Galileo reasoned that by knowing the transit time of the light beams from one lantern to the other and the distance between the two lanterns, he could obtain the speed. His results were inconclusive. Today, we realize (as Galileo concluded) that it is impossible to measure the speed of light in this manner because the transit time for the light is so much less than the reaction time of the observers.

2.1. Roemer's Method

- In 1675, Danish astronomer Ole Roemer (1644–1710) made the first successful estimate of the speed of light. Ole Roemer sought to determine the velocity of light by observing even further distances. Roemer studied the eclipses of Jupiter's moon, I_0 .
- Roemer timed the moment that I_0 became hidden by Jupiter and then the moment that it reappeared. He would then compare this

time interval as Earth moved closer and further away from Jupiter. Roemer realized that as Earth moved away from Jupiter, the time interval for time spent hidden increased. As earth rotated back closer to Jupiter, the hidden time interval would slowly decrease.

- Although Roemer himself never actually gave a numerical value for the speed of light, he did state that light requires about a second (s) of time to traverse a distance equal to the diameter of the earth. From this statement it can be deduced that light would require 22 minutes to traverse the earth's orbit.
- These numbers were combined by Christian Huygens to give the value of 2.20×10^8 m/s which gives a percent error of about 26% from the currently accepted value.
- Although the value is inaccurate, it was the first reproducible result for the speed of light and gave proof that the speed of light was finite. (Fig.1).

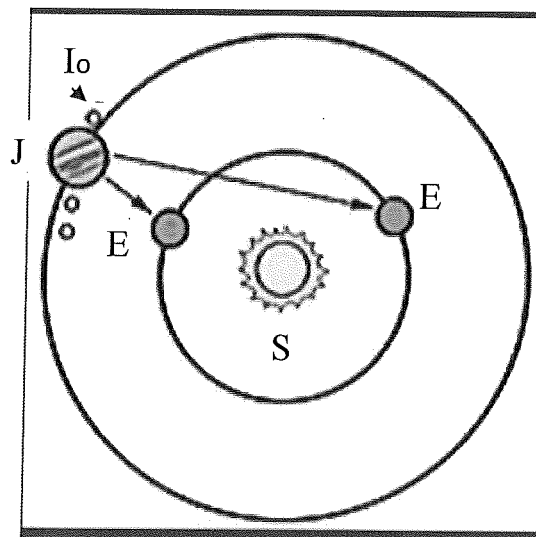


Figure .1 Roemer's method for measuring the speed of light.

2.2. Fizeau's Method

The first successful method for measuring the speed of light was developed in 1849 by French physicist Armand H. L. Fizeau (1819–1896).

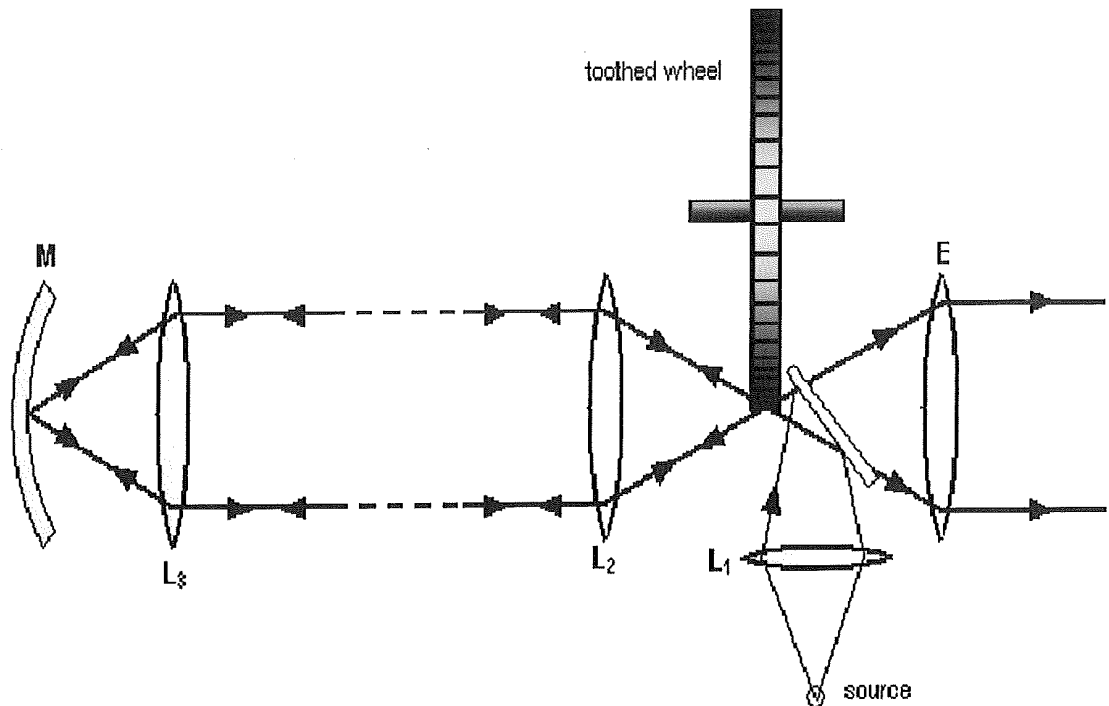


Figure.2 Fizeau's method for measuring the speed of light using a rotating toothed wheel.

- Light from a source is focused onto a rotating toothed wheel by reflection from a glass slide.
- Light passes through a gap between the teeth of the wheel.
- The reflected light is focused by mirrors L_2 and L_3 onto a mirror M , then back onto the toothed wheel.
- The speed of rotation of the wheel is adjusted so that the time for the light to pass from the wheel to M and back is the same as the time taken for a tooth to move into the gap through which the light passed.

- If the distance from the wheel to M is d , then in the time taken for a tooth to move into the gap, the light has travelled a distance $2d$. If the wheel has N teeth, radius r and is rotating at n revolutions per second, then the tooth has travelled a fraction $1/2N$ of the circumference of the wheel at a speed of $n \times 2\pi r$ so

$$t = \frac{\frac{1}{2N} \times 2\pi r}{n \times 2\pi r} = \frac{1}{2nN}$$

Applying the relation
gives

$$\text{speed} = \frac{\text{distance}}{\text{time}} \quad \text{for light}$$

$$c = \frac{2d}{1/2nN} = 4dnN$$

Fizeau used

$$d = 8630 \text{ m},$$

$$N = 720,$$

$$n = 12.6 \text{ revs/sec}, \text{ giving}$$

$$\text{giving} \quad C = 4dnN = 3.13 \times 10^8 \text{ m/s}$$

Although the idea of the experiment is quite simple, there were some disadvantages of the method due to the following reasons:

- 1- The complete dark or disappearance of light cannot be obtained due to scattering of light from the teeth.

- 2- The image of the source is very faint because the intensity of light is considerably decreased due to refraction and reflection at various surfaces of the lenses and mirrors.
- 3- The uniform speed of rotation of the wheel cannot be attained.

Example 1: Measuring the Speed of Light with Fizeau's Wheel

Assume Fizeau's wheel has 360 teeth and rotates at 27.5 rev/s when a pulse of light passing through the gap between two teeth. If the distance to the mirror is 7 500 m, what is the speed of light?

Solution

Applying the equation $C = 4dnN$

Then

$$\begin{aligned} C &= 4 \times 7500 \text{ m} \times 27.5 \text{ rev/s} \times 360 \\ &= 2.97 \times 10^8 \text{ m/s} \end{aligned}$$

3- The Ray Approximation in Geometric Optics

The field of **geometric optics** involves the study of the propagation of light. Geometric optics assumes light travels in a fixed direction in a straight line as it passes through a uniform medium and changes its direction when it meets the surface of a different medium or if the optical properties of the medium are nonuniform in either space or time. In our study of geometric optics here, we use what is called the **ray approximation**. To understand this approximation, first notice that the rays of a given wave are straight lines perpendicular to the wave fronts as illustrated in (Figure.3) for a plane wave. In the ray approximation, a wave moving through a medium travels in a straight line in the direction of its rays.

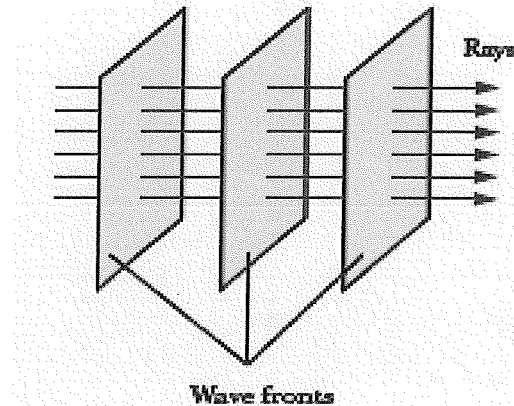


Figure.3 A plane wave propagating to the right. Notice that the rays, which always point in the direction of the wave propagation, are straight lines perpendicular to the wave fronts.

- If the wave meets a barrier in which there is a circular opening whose diameter is much larger than the wavelength as in Active (Figure.4a), the wave emerging from the opening continues to move in a straight line; hence, the ray approximation is valid.

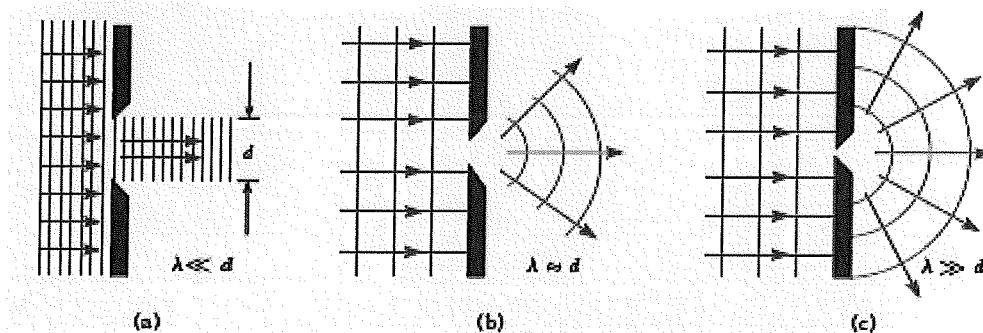


Figure.4 A plane wave of wavelength λ is incident on a barrier in which there is an opening of diameter d . (a) When $\lambda \ll d$, the rays continue in a straight-line path and the ray approximation remains valid. (b) When $\lambda \approx d$, the rays spread out after passing through the opening. (c) When $\lambda \gg d$, the opening behaves as a point source emitting spherical waves.

- If the diameter of the opening is on the order of the wavelength as in Active (Figure.4b), the waves spread out from the opening in all directions. This effect, called **diffraction**, will be studied later.

- Finally, if the opening is much smaller than the wavelength, the opening can be approximated as a point source of waves as shown in Active (Fig.4c).

4- The Wave Under Reflection

- When a light ray traveling in one medium encounters a boundary with another medium, part of the incident light is reflected. For waves on a one-dimensional string, the reflected wave must necessarily be restricted to a direction along the string.

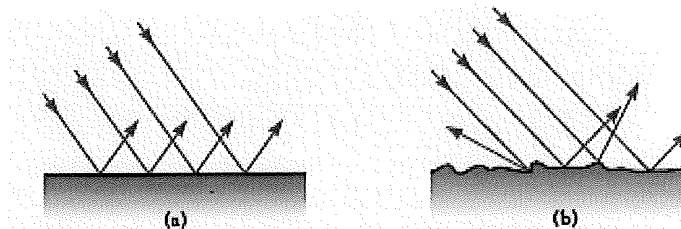


Figure.5 Schematic representation of (a) specular reflection, where the reflected rays are all parallel to each other, and (b) diffuse reflection, where the reflected rays travel in random directions.

- For light waves traveling in three dimensional space, no such restriction applies and the reflected light waves can be in directions different from the direction of the incident waves.
- Figure 5a shows several rays of a beam of light incident on a smooth, mirror-like, reflecting surface.
- The reflected rays are parallel to one another as indicated in the figure. The direction of a reflected ray is in the plane perpendicular to the reflecting surface that contains the incident ray.

- Reflection of light from such a smooth surface is called **specular reflection**. If the reflecting surface is rough as in Figure 5b, the surface reflects the rays not as a parallel set but in various directions.
- Reflection from any rough surface is known as **diffuse reflection**. A surface behaves as a smooth surface as long as the surface variations are much smaller than the wavelength of the incident light.
- The difference between these two kinds of reflection explains why it is more difficult to see while driving on a rainy night than on a dry, sunny day. If the road is wet, the smooth surface of the water specularly reflects most of your headlight beams away from your car (and perhaps into the eyes of oncoming drivers). When the road is dry, its rough surface diffusely reflects part of your headlight beam back toward you, allowing you to see the road more clearly. In this book, we restrict our study to specular reflection and use the term *reflection* to mean specular reflection.
- Consider a light ray traveling in air and incident at an angle on a flat, smooth surface as shown in Active Figure 6. The incident and reflected rays make angles θ_1 and θ'_1 , respectively, where the angles are measured between the normal and the rays. (The normal is a line drawn perpendicular to the surface at the point where the incident ray strikes the surface.) Experiments and theory show that **the angle of reflection equals the angle of incidence:**

$$\theta'_1 = \theta_1 \quad (2)$$

- This relationship is called the **law of reflection**. Because reflection of waves from an interface between two media is a common phenomenon, we identify an analysis model for this situation: the **wave under reflection**. Equation.2 is the mathematical representation of this model.

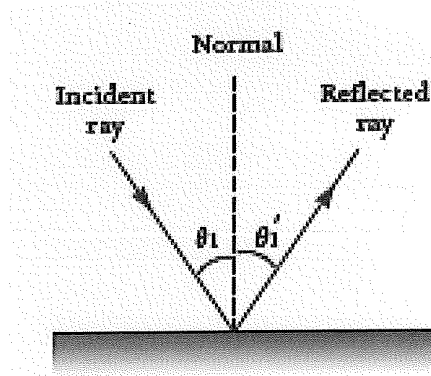


FIGURE.6 According to the wave under reflection model, $\theta'_1 = \theta_1$. The incident ray, the reflected ray, and the normal all lie in the same plane.

Example 2 The Double-Reflected Light Ray

Two mirrors make an angle of 120° with each other as illustrated in Figure 7a. A ray is incident on mirror M_1 at an angle of 65° to the normal. Find the direction of the ray after it is reflected from mirror M_2 .

Solution

From the law of reflection, the first reflected ray makes an angle of 65° with the normal. Find the angle the first reflected ray makes with the horizontal:

$$\delta = 90^\circ - 65^\circ = 25^\circ$$

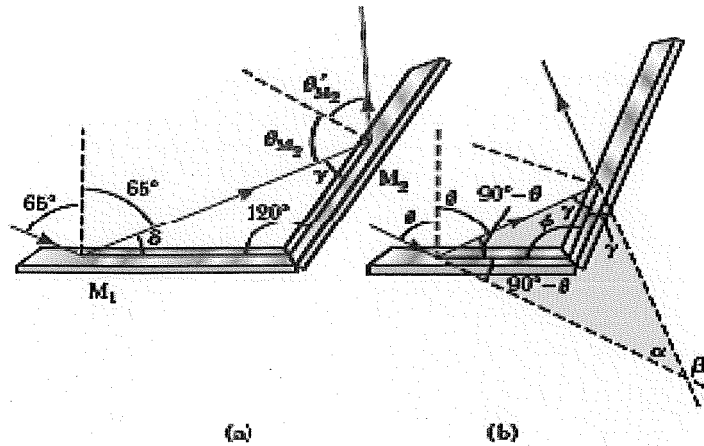


Figure .7 (Example.2) (a) Mirrors M_1 and M_2 make an angle of 120° with each other. (b) The geometry for an arbitrary mirror angle.

From the triangle made by the first reflected ray and the two mirrors, find the angle the reflected ray makes with M_2 :

$$\gamma = 180^\circ - 25^\circ - 120^\circ = 35^\circ$$

Find the angle the first reflected ray makes with the normal to M_2 :

$$\theta_{M_2} = 90^\circ - 35^\circ = 55^\circ$$

From the law of reflection, find the angle the second reflected ray makes with the normal to M_2

$$\theta'_{M_2} = \theta_{M_2} = 55^\circ$$

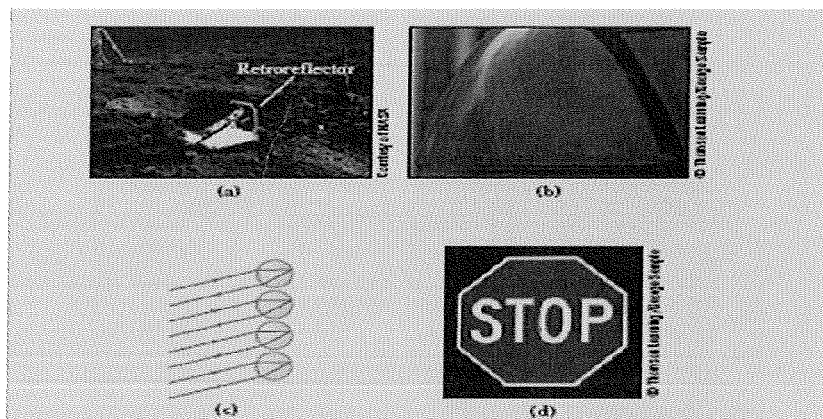


Figure .8 Applications of retroreflection.

- If the angle between two mirrors is 90° , the reflected beam returns to the source parallel to its original path. This phenomenon, called **retroreflection**, has many practical applications.
- If a third mirror is placed perpendicular to the first two so that the three form the corner of a cube, retroreflection works in three dimensions. In 1969, a panel of many small reflectors was placed on the Moon by the *Apollo 11* astronauts (Fig.8a). A laser beam from the Earth is reflected directly back on itself, and its transit time is measured. This information is used to determine the distance to the Moon with an uncertainty of 15 cm.
- A more everyday application is found in automobile taillights. Part of the plastic making up the taillight is formed into many tiny cube corners (Fig.8b) so that headlight beams from cars approaching from the rear are reflected back to the drivers.
- Instead of cube corners, small spherical bumps are sometimes used (Fig.8c). Tiny clear spheres are used in a coating material found on many road signs. Due to retroreflection from these spheres, the stop sign in (Fig.8d) appears much brighter than it would if it were simply a flat, shiny surface.
- Retroreflectors are also used for reflective panels on running shoes and running clothing to allow joggers to be seen at night.

5. The Wave Under Refraction

In addition to the phenomenon of reflection for waves, it was also found that some of the energy of the incident wave transmits into the

new medium. Similarly, when a ray of light traveling through a transparent medium encounters a boundary leading into another transparent medium as shown in Figure.9, part of the energy is reflected and part enters the second medium. As with reflection, the direction of the transmitted wave exhibits an interesting behavior because of the three-dimensional nature of the light waves. The ray that enters the second medium is bent at the boundary and is said to be **refracted**.

- The incident ray, the reflected ray, and the refracted ray all lie in the same plane. The **angle of refraction**, θ_2 in Active Figure 9, depends on the properties of the two media and on the angle of incidence θ_1 through the relationship

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} \quad (3)$$

where v_1 is the speed of light in the first medium and v_2 is the speed of light in the second medium.

- The path of a light ray through a refracting surface is reversible. For example, the ray shown in Active Fig.9 travels from point *A* to point *B*. If the ray originated at *B*, it would travel along line *BA* to reach point *A* and the reflected ray would point downward and to the left in the glass.
- From Equation.3, we can infer that when light moves from a material in which its speed is high to a material in which its speed is lower.

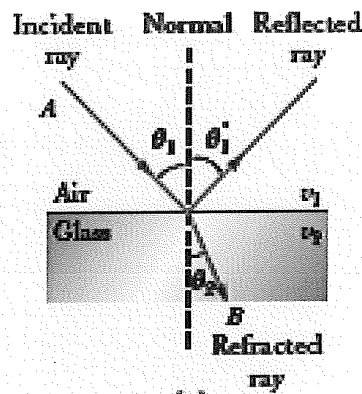


FIGURE .9 A ray obliquely incident on an air–glass interface behaves according to the wave under refraction model. The refracted ray is bent toward the normal because $v_2 < v_1$. All rays and the normal lie in the same plane.

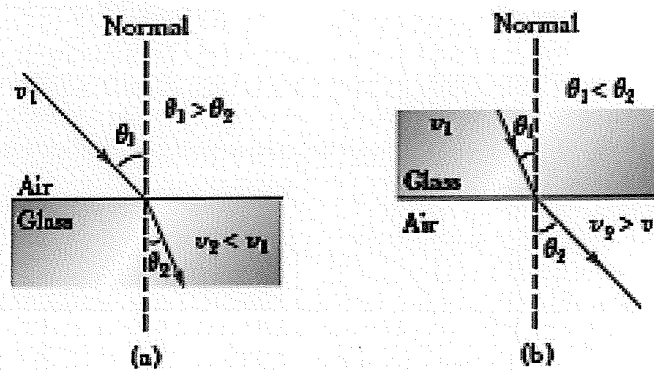


FIGURE.10 (a) When the light beam moves from air into glass, the light slows down upon entering the glass and its path is bent toward the normal. (b) When the beam moves from glass into air, the light speeds up upon entering the air and its path is bent away from the normal.

- Figure.10a, the angle of refraction θ_2 is less than the angle of incidence θ_1 and the ray is bent *toward* the normal.
- If the ray moves from a material in which light moves slowly to a material in which it moves more rapidly as illustrated in Active

Figure.10b, θ_2 is greater than θ_1 and the ray is bent *away* from the normal.

- The behavior of light as it passes from air into another substance and then reemerges into air is often a source of confusion to students. When light travels in air, its speed is 3.00×10^8 m/s, but this speed is reduced to approximately 2×10^8 m/s when the light enters a block of glass. When the light re-emerges into air, its speed instantaneously increases to its original value of 3.00×10^8 m/s. This effect is far different from what happens, for example, when a bullet is fired through a block of wood. In that case, the speed of the bullet decreases as it moves through the wood because some of its original energy is used to tear apart the wood fibers. When the bullet enters the air once again, it emerges at a speed lower than it had when it entered the wood.

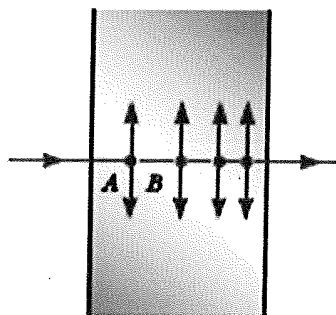


Figure.11 Light passing from one atom to another in a medium. The dots are electrons, and the vertical arrows represent their oscillations.

- To see why light behaves as it does, consider Figure.11, which represents a beam of light entering a piece of glass from the left. Once inside the glass, the light may encounter an electron bound

to an atom, indicated as point *A*. Let's assume light is absorbed by the atom, which causes the electron to oscillate.

- The oscillating electron then acts as an antenna and radiates the beam of light toward an atom at *B*, where the light is again absorbed.
- For now, it is sufficient to think of light passing from one atom to another through the glass. Although light travels from one atom to another at 3.00×10^8 m/s, the absorption and radiation that take place cause the *average* light speed through the material to fall to about 2×10^8 m/s. Once the light emerges into the air, absorption and radiation cease and the light travels at a constant speed of 3.00×10^8 m/s.

Index of Refraction

In general, the speed of light in any material is less than its speed in vacuum. In fact, light travels at its maximum speed c in vacuum. It is convenient to define the index of refraction n of a medium to be the ratio

$$n \equiv \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} \equiv \frac{c}{v} \quad (4)$$

This definition shows that the index of refraction is a dimensionless number greater than unity because v is always less than c . Furthermore, n is equal to unity for vacuum. The indices of refraction for various substances are listed in Table.1. As light travels from one medium to another, its frequency does not change but its wavelength does. To see why that is true, consider Fig.12

Table 1

Indices of Refraction			
Substance	Index of Refraction	Substance	Index of Refraction
<i>Solids at 20°C</i>		<i>Liquids at 20°C</i>	
Cubic zirconia	2.20	Benzene	1.501
Diamond (C)	2.419	Carbon disulfide	1.628
Fluorite (CaF ₂)	1.434	Carbon tetrachloride	1.461
Fused quartz (SiO ₂)	1.458	Ethyl alcohol	1.361
Gallium phosphide	3.50	Glycerin	1.473
Glass, crown	1.52	Water	1.333
Glass, flint	1.66		
Ice (H ₂ O)	1.309	<i>Gases at 0°C, 1 atm</i>	
Polystyrene	1.49	Air	1.000 293
Sodium chloride (NaCl)	1.544	Carbon dioxide	1.000 45

Note: All values are for light having a wavelength of 589 nm in vacuum.

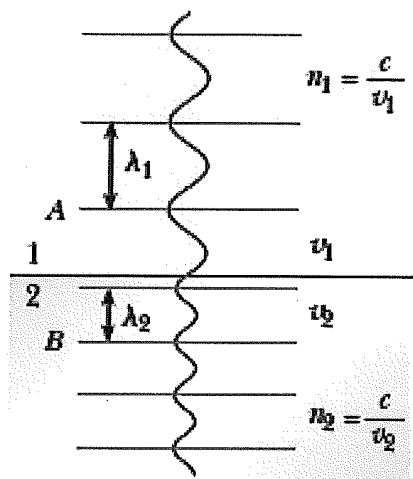


Figure .12 As a wave moves from medium 1 to medium 2, its wavelength changes but its frequency

Waves pass an observer at point A in medium 1 with a certain frequency and are incident on the boundary between medium 1 and medium 2. The frequency with which the waves pass an observer at point B in medium 2 must equal the frequency at which they pass point A. If that were not the case, energy would be piling up or

disappearing at the boundary. Because there is no mechanism for that to occur, the frequency must be a constant as a light ray passes from one medium into another. Therefore, because the relationship $v = \lambda f$ must be valid in both media and because $f_1 = f_2 = f$, we see that

$$v_1 = l_1 f \quad \text{and} \quad v_2 = l_2 f \quad (5)$$

Because $v_1 \neq v_2$, it follows that $\lambda_1 \neq \lambda_2$ as shown in Fig.12, We can obtain a relationship between index of refraction and wavelength by dividing the first Equ.5 by the second and then using Equ.4:

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1} \quad (6)$$

This expression gives

$$\lambda_1 n_1 = \lambda_2 n_2$$

If medium 1 is vacuum or, for all practical purposes, air, then $n_1 = 1$. Hence, it follows from Equ.6 that the index of refraction of any medium can be expressed as the ratio

$$n = \frac{\lambda}{\lambda_n} \quad (7)$$

where λ is the wavelength of light in vacuum and λ_n is the wavelength of light in the medium whose index of refraction is n . From Equation.7, we see that because $n > 1$, $\lambda_n < \lambda$.

We are now in a position to express Equation.3 in an alternative form. Replacing the v_2/v_1 term in Equation.3 with n_1/n_2 from Equation.6 gives

$$\boxed{n_1 \sin \theta_1 = n_2 \sin \theta_2} \quad (8)$$

Quick Quiz 1 Light passes from a material with index of refraction 1.3 into one with index of refraction 1.2. Compared to the incident ray, what happens to the refracted ray? (a) It bends toward the normal. (b) It is undeflected. (c) It bends away from the normal.

The index of refraction is inversely proportional to the wave speed. As the wave speed v decreases, the index of refraction n increases. Therefore, the higher the index of refraction of a material, the more it slows down light from its speed in vacuum. The more the light slows down, the more θ_2 differs from θ_1 in Equation.8

EXAMPLE .3

A light ray of wavelength 589 nm traveling through air is incident on a smooth, flat slab of crown glass at an angle of 30.0° to the normal. Knowing that $n_1 = 1.00$ for air and $n_2 = 1.52$ for crown glass: (A) Find the angle of refraction. (B) Find the speed of this light once it enters the glass. (C) What is the wavelength of this light in the glass?

SOLUTION

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1$$

$$\sin \theta_2 = \left(\frac{1.00}{1.52} \right) \sin 30.0^\circ = 0.329$$

$$\begin{aligned} \theta_2 &= \sin^{-1}(0.329) \\ &= 19.2^\circ \end{aligned}$$

$$v = \frac{c}{n}$$

$$v = \frac{3.00 \times 10^8 \frac{m}{s}}{1.52} = 1.97 \times 10^8 \frac{m}{s}$$

$$\lambda_n = \frac{\lambda}{n} = \frac{589 \text{ nm}}{1.52} = 388 \text{ nm}$$

EXAMPLE .4 Light Passing Through a Slab

A light beam passes from medium 1 to medium 2, with the latter medium being a thick slab of material whose index of refraction is n_2 (Fig.13). Show that the beam emerging into medium 1 from the other side is parallel to the incident beam

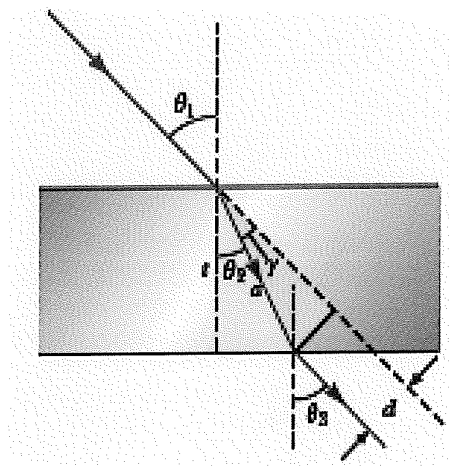


Figure .13 When light passes through a flat slab of material, the emerging beam is parallel to the incident beam; therefore, $\theta_1 = \theta_3$. The dashed line drawn parallel to the ray coming out the bottom of the slab represents the path the light would take were the slab not there.

Solution

$$(1) \quad \sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1$$

$$(2) \quad \sin \theta_3 = \frac{n_2}{n_1} \sin \theta_2$$

$$\sin \theta_3 = \frac{n_2}{n_1} \left(\frac{n_1}{n_2} \sin \theta_1 \right) = \sin \theta_1$$

What If? What if the thickness t of the slab is doubled? Does the offset distance d also double?

Answer Consider the region of the light path within the slab in Fig.13. The distance a is the hypotenuse of two right triangles.

$$a = \frac{t}{\cos \theta_2}$$

$$d = a \sin \gamma = a \sin (\theta_1 - \theta_2)$$

$$d = \frac{t}{\cos \theta_2} \sin (\theta_1 - \theta_2)$$

For a given incident angle θ_1 , the refracted angle θ_2 is determined solely by the index of refraction, so the offset distance d is proportional to t . If the thickness doubles, so does the offset distance.

In Example.4

- What happens when light strikes a prism with nonparallel sides as shown in Figure.14?
- In this case, the outgoing ray does not propagate in the same direction as the incoming ray.
- A ray of single-wavelength light incident on the prism from the left emerges at angle δ from its original direction of travel. This angle δ is called the angle of deviation.
- The apex angle φ of the prism, shown in the figure, is defined as: the angle between the surface at which the light enters the prism and the second surface that the light encounters.

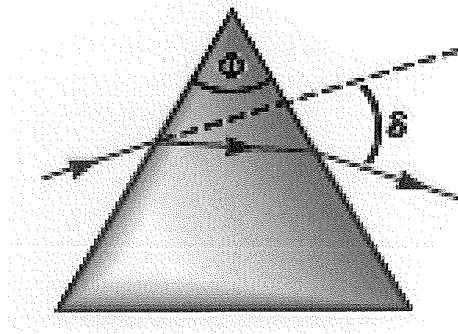


Figure 14 A prism refracts a single-wavelength light ray through an angle of deviation δ .

EXAMPLE 5. Measuring n Using a Prism

Although we do not prove it here, the minimum angle of deviation $\delta_{\min}$ for a prism occurs when the angle of incidence θ_1 is such that the refracted ray inside the prism makes the same angle with the normal to the two prism faces as shown in Figure.15. Obtain an expression for the index of refraction of the prism material in terms of the minimum angle of deviation $\delta_{\min}$ and the apex angle Φ .

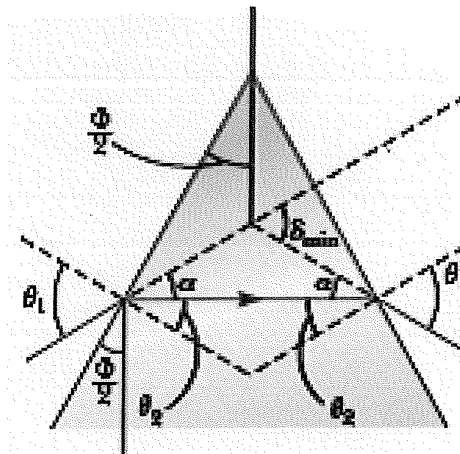


Figure .15 A light ray passing through a prism at the minimum angle of deviation $\delta_{\min}$.

SOLUTION

In this example, light enters a material through one surface and leaves the material at another surface. Let's apply the wave under refraction model at each surface.

Analyze Consider the geometry in Figure.15.

$$\theta_1 = \alpha + \theta_2 \quad (1)$$

$$\delta_{min} = 2\alpha \quad (2)$$

$$\theta_2 = \frac{\varphi}{2} \quad (3)$$

Substituting from (2) & (3) in (1) we get:

$$\theta_1 = \left(\frac{\delta_{min} + \varphi}{2} \right) \quad (4)$$

As : $n_1 \sin \theta_1 = n_2 \sin \theta_2$

In case $n_1 = 1$ & $n_2 = n$ then,

$$\sin \theta_1 = n \sin \theta_2 \quad \rightarrow \rightarrow \rightarrow \rightarrow \quad n = \frac{\sin \theta_1}{\sin \theta_2} \quad (5)$$

From (4) & (3) in (5) then we get:

$$n = \frac{\sin \left(\delta_{min} + \varphi/2 \right)}{\sin \varphi/2} \quad (9)$$

Knowing the apex angle Φ of the prism and measuring δ_{min} , you can calculate the index of refraction of the prism material. Furthermore, a hollow prism can be used to determine the values of n for various liquids filling the prism.

6. Huygens's Principle

In this section, we develop the laws of reflection and refraction by using a geometric method proposed by Huygens in 1678. Huygens's principle is a geometric construction for using knowledge of an earlier

wave front to determine the position of a new wave front at some instant. In Huygens's construction, *all points on a given wave front are taken as point sources for the production of spherical secondary waves, called wavelets, that propagate outward through a medium with speeds characteristic of waves in that medium. After some time interval has passed, the new position of the wave front is the surface tangent to the wavelets.*

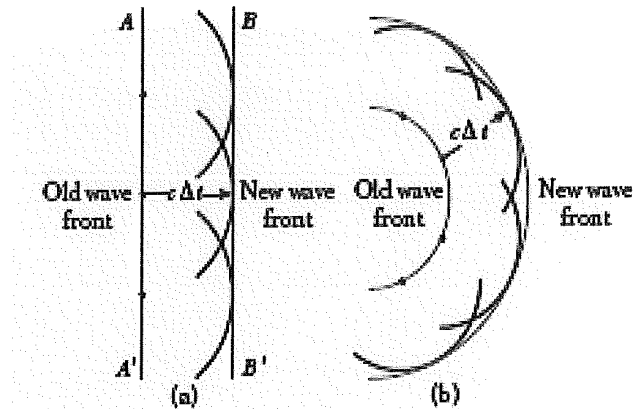


Figure .16 Huygens's construction for (a) a plane wave propagating to the right and (b) a spherical wave propagating to the right.

First, consider a plane wave moving through free space as shown in Figure.16a. At $t = 0$, the wave front is indicated by the plane labeled AA'. In Huygens's construction, each point on this wave front is considered a point source. For clarity, only three points on AA' are shown. With these points as sources for the wavelets, we draw circles, each of radius $c \Delta t$, where c is the speed of light in vacuum and Δt is some time interval during which the wave propagates. The surface drawn tangent to these wavelets is the plane BB' which is the wave front at a later

6.1. Huygens's Principle Applied to Reflection and Refraction

- The laws of reflection and refraction were stated earlier in this chapter without proof. We now derive these laws, using Huygens's principle.
- For the law of reflection, refer to Figure.17. The line AB represents a plane wave front of the incident light just as ray 1 strikes the surface. At this instant, the wave at A sends out a Huygens wavelet (the circular arc centered on A). The reflected light propagates toward D.
- At the same time, the wave at B emits a Huygens wavelet (the circular arc centered on B) with the light propagating toward C. Figure.17 shows these wavelets after a time interval Δt , after which ray 2 strikes the surface. Because both rays 1 and 2 move with the same speed, we must have $AD = BC = c \Delta t$.

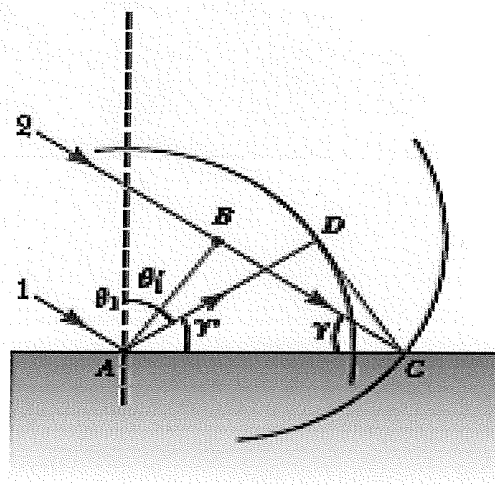


Figure.17 Huygens's construction for proving the law of reflection.

- Depends on geometry, notice that the two triangles ABC and ADC are congruent because they have the same hypotenuse AC and because $AD = BC$. Figure.17 shows that:

$$\cos \gamma = \frac{BC}{AC} \quad \text{and} \quad \cos \gamma' = \frac{AD}{AC}$$

Where $\gamma = 90^\circ - \theta_1$ and $\gamma' = 90^\circ - \theta'_1$, because $AD = BC$

Therefore:

$$\gamma = \gamma'$$

$$90^\circ - \theta_1 = 90^\circ - \theta'_1$$

and

$$\boxed{\theta_1 = \theta'_1}$$

which is the law of reflection.

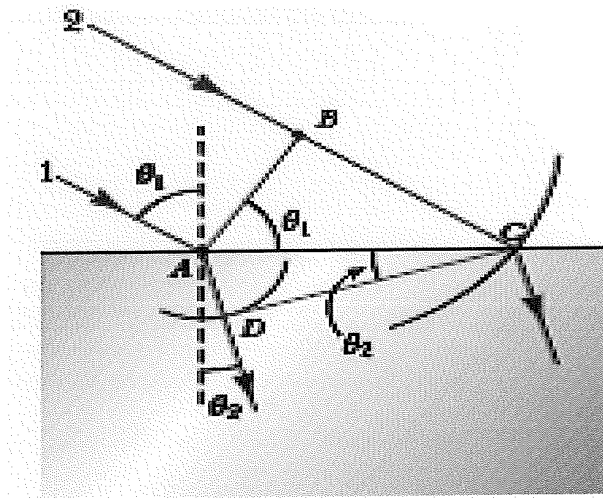


Figure.18 Huygens's construction for proving Snell's law of refraction.

- Now let's use Huygens's principle and Figure18 to derive Snell's law of refraction.
- We focus our attention on the instant ray 1 strikes the surface and the subsequent time interval until ray 2 strikes the surface. During this time interval, the wave at A sends out a Huygens wavelet (the arc centered on A) and the light refracts toward D.
- In the same time interval, the wave at B sends out a Huygens wavelet (the arc centered on B) and the light continues to propagate toward C.

- Because these two wavelets travel through different media, the radii of the wavelets are different. The radius of the wavelet from A is $AD = v_2 \Delta t$, where v_2 is the wave speed in the second medium. The radius of the wavelet from B is $BC = v_1 \Delta t$, where v_1 is the wave speed in the original medium.
- From triangle ABC and ADC, we find that

$$\sin \theta_1 = \frac{BC}{AC} = \frac{v_1 \Delta t}{AC} \quad \text{and} \quad \sin \theta_2 = \frac{AD}{AC} = \frac{v_2 \Delta t}{AC}$$

Dividing the first equation by the second gives

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2}$$

From equation (4) we know that $v_1 = c/n_1$ and $v_2 = c/n_2$ therefore,

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

and

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

which is Snell's law of refraction

7- Dispersion

An important property of the index of refraction n is that, for a given material, the index varies with the wavelength of the light passing through the material as Figure.19 shows. This behavior is called **dispersion**. Because n is a function of wavelength, Snell's law of refraction indicates that light of different wavelengths is refracted at different angles when incident on a material. Figure.19 shows that the index of refraction generally decreases with increasing wavelength. For example, violet light refracts more than red light does when passing into a material.

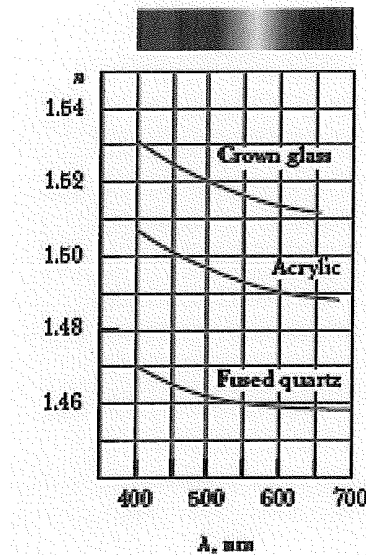


Figure.19 Variation of index of refraction with vacuum wavelength for three materials.

- Now suppose a beam of white light (a combination of all visible wavelengths) is incident on a prism as illustrated in Figure.20.

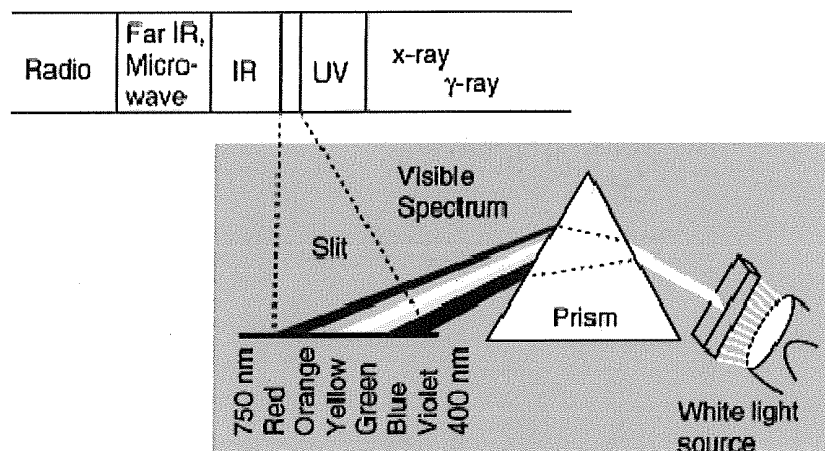


Figure.20 White light enters a glass prism at the upper left.

- Clearly, the angle of deviation δ depends on wavelength. The rays that emerge spread out in a series of colors known as the visible spectrum. These colors, in order of decreasing wavelength, are red, orange, yellow, green, blue, and violet.

- Newton showed that each color has a particular angle of deviation and that the colors can be recombined to form the original white light.

8 - Total Internal Reflection

An interesting effect called total internal reflection can occur when light is directed from a medium having a given index of refraction toward one having a lower index of refraction. Consider Fig.21a, in which a light ray travels in medium 1 and meets the boundary between medium 1 and medium 2, where n_1 is greater than n_2 .

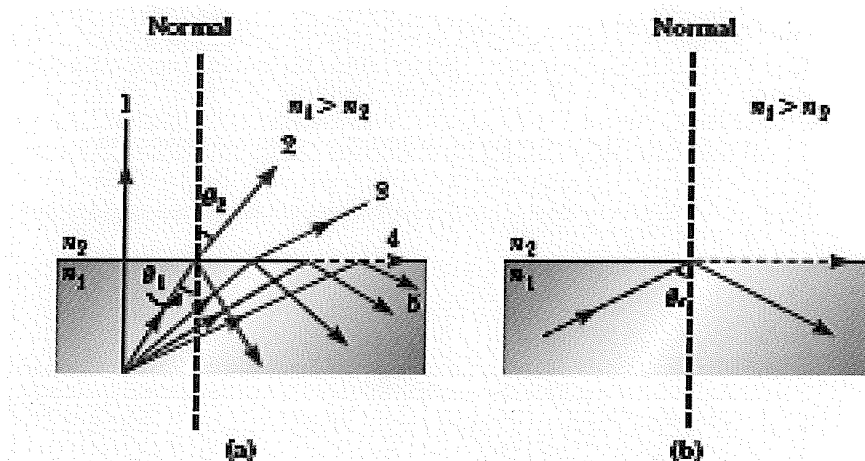


FIGURE.21 (a) Rays travel from a medium of index of refraction n_1 into a medium of index of refraction n_2 , where $n_2 < n_1$. (b) The angle of incidence producing an angle of refraction equal to 90° is the critical angle θ_c .

- In the figure, labels 1 through 5 indicate various possible directions of the ray consistent with the wave under refraction model. The refracted rays are bent away from the normal because n_1 is greater than n_2 .

- At some particular angle of incidence θ_c , called the critical angle, the refracted light ray moves parallel to the boundary so that $\theta_2 = 90^\circ$ (Fig.21b). For angles of incidence greater than θ_c , the ray is entirely reflected at the boundary as shown by ray 5 in Figure.21a.
- We can use Snell's law of refraction to find the critical $\theta_2 = 90^\circ$ and Equation 8 gives

$$n_1 \sin \theta_c = n_2 \sin 90^\circ = n_2$$

$$\sin \theta_c = \frac{n_2}{n_1} \quad (\text{for } n_1 > n_2) \quad (10)$$

- This equation can be used only when n_1 is greater than n_2 . That is, total internal reflection occurs only when light is directed from a medium of a given index of refraction toward a medium of lower index of refraction. If n_1 were less than n_2 , Equation.10 would give $\sin \theta_c = 1$, which is a meaningless result because the sine of an angle can never be greater than unity.
- The critical angle for total internal reflection is small when n_1 is considerably greater than n_2 . For example, the critical angle for a diamond in air is 24° . Any ray inside the diamond that approaches the surface at an angle greater than 24° is completely reflected back into the crystal. This property, combined with proper faceting, causes diamonds to sparkle. The angles of the facets are cut so that light is "caught" inside the crystal through multiple internal reflections. These multiple reflections give the light a long path through the medium, and substantial dispersion

of colors occurs. By the time the light exits through the top surface of the crystal, the rays associated with different colors have been fairly widely separated from one another.

EXAMPLE .6: A View from the Fish's Eye

Find the critical angle for an air–water boundary. (The index of refraction of water is 1.33.)

SOLUTION

Study Figure 22 to understand the concept of total internal reflection and the significance of the critical angle. Apply Equation 10 to the air–water interface:

$$\sin \theta_c = \frac{n_2}{n_1} = \frac{1.00}{1.33} = 0.752$$
$$\theta_c = 48.8^\circ$$

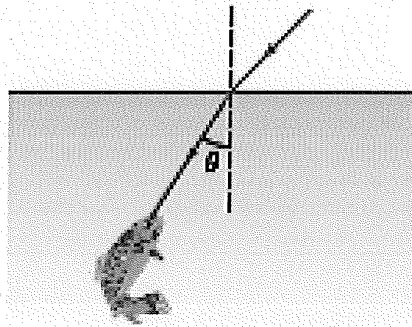


Figure .22 A fish looks upward toward the water surface

What If ? What if a fish in a still pond looks upward toward the water's surface at different angles relative to the surface as in Figure.22? What does it see?

Answer : Because the path of a light ray is reversible, light traveling from medium 2 into medium 1 in Figure.21a follows the paths shown, but in the opposite direction. A fish looking upward toward the water

surface as in Figure.22 can see out of the water if it looks toward the surface at an angle less than the critical angle. Therefore, when the fish's line of vision makes an angle of $\theta = 40^\circ$ with the normal to the surface, for example, light from above the water reaches the fish's eye. At $\theta = 48.8^\circ$, the critical angle for water, the light has to skim along the water's surface before being refracted to the fish's eye; at this angle, the fish can, in principle, see the entire shore of the pond. At angles greater than the critical angle, the light reaching the fish comes by means of total internal reflection at the surface. Therefore, at $\theta = 60^\circ$, the fish sees a reflection of the bottom of the pond.

Optical Fibers

Another interesting application of total internal reflection is the use of glass or transparent plastic rods to "pipe" light from one place to another. As indicated in Figure.23, light is confined to traveling within a rod, even around curves, as the result of successive total internal reflections. Such a light pipe is flexible if thin fibers are used rather than thick rods. A flexible light pipe is called an optical fiber. If a bundle of parallel fibers is used to construct an optical transmission line, images can be transferred from one point to another. This technique is used in a sizable industry known as fiber optics.

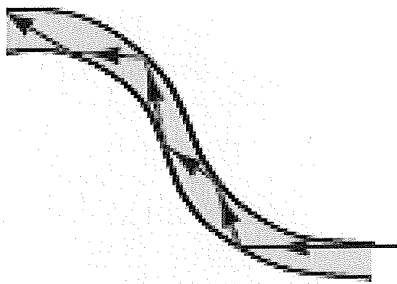


Figure.23 Light travels in a curved transparent rod by multiple internal reflections.

A practical optical fiber consists of a transparent core surrounded by a cladding, a material that has a lower index of refraction than the core. The combination may be surrounded by a plastic jacket to prevent mechanical damage. Because the index of refraction of the cladding is less than that of the core, light traveling in the core experiences total internal reflection if it arrives at the interface between the core and the cladding at an angle of incidence that exceeds the critical angle. In this case, light “bounces” along the core of the optical fiber, losing very little of its intensity as it travels.

Chapter 2

Image Formation

This chapter is concerned with the images that result when light rays encounter flat and curved surfaces. Images can be formed by either reflection or refraction, and we can design mirrors and lenses to form images with desired characteristics. We continue to use the ray approximation and assume light travels in straight lines. These two steps lead to valid predictions in the field called *geometric optics*.

1- Images Formed by Flat Mirrors

Image formation by mirrors can be understood through the analysis of light rays following the wave under reflection model.

- We begin by considering the simplest possible mirror, the flat mirror. Consider a point source of light placed at O in Figure .1, a distance p in front of a flat mirror. The distance p is called the **object distance**. Diverging light rays leave the source and are reflected from the mirror.
- Upon reflection, the rays continue to diverge. The dashed lines in Figure.1 are extensions of the diverging rays back to a point of intersection at I .
- The diverging rays appear to the viewer to originate at the point I behind the mirror. Point I , which is a distance q behind the mirror, is called the **image** of the object at O . The distance q is called the **image distance**.

- Images can always be located by extending diverging rays back to a point at which they intersect. Images are located either at a point from which rays of light actually diverge or at a point from which they appear to diverge.
- Images are classified as real or virtual.
 - 1- A real image is formed when light rays pass through and diverge from the image point. Real images can be displayed on a screen (as at a movie theater),
 2. A virtual image is formed when the light rays do not pass through the image point but only appear to diverge from that point. Virtual images cannot be displayed on a screen. The image formed by the mirror in Figure.1 is virtual. The image of an object seen in a flat mirror is always virtual.

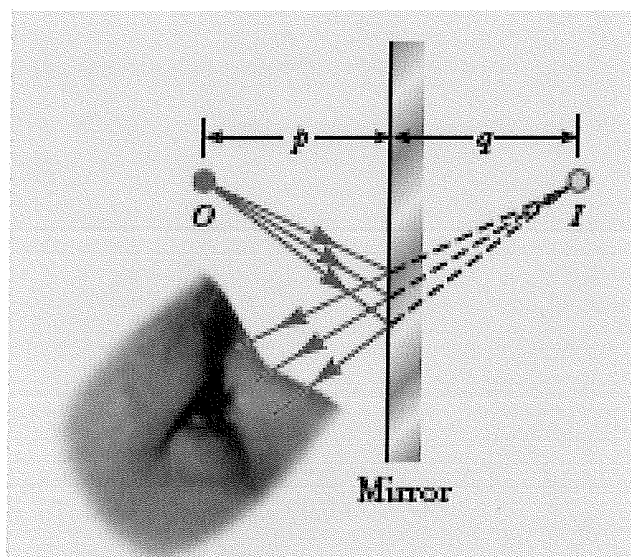


Figure .1 An image formed by reflection from a flat mirror.

- We can use the simple geometry in Figure.2 to examine the properties of the images of extended objects formed by flat mirrors.

- Even though there are an infinite number of choices of directions in which light rays could leave each point on the object, we need to choose only two rays to determine where an image is formed. One of those rays starts at P, follows a path perpendicular to the mirror, and reflects back on itself. The second ray follows the oblique path PR and reflects as shown in Figure.2 according to the law of reflection.

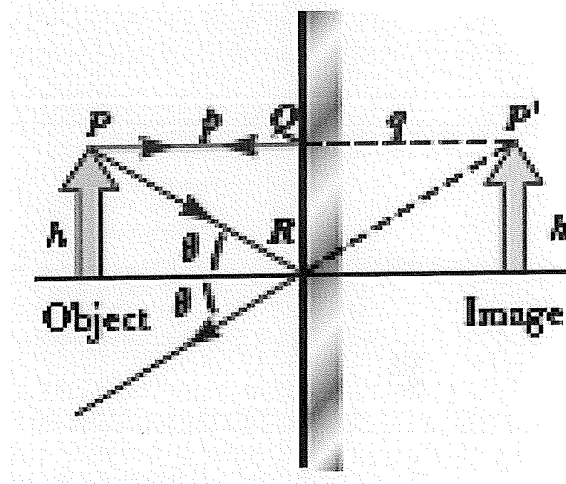


FIGURE .2 A geometric construction that is used to locate the image of an object placed in front of a flat mirror.

- An observer in front of the mirror would extend the two reflected rays back to the point at which they appear to have originated, which is point P' behind the mirror.
- A continuation of this process for points other than P on the object would result in a virtual image of the entire object behind the mirror.
- Because triangles PQR and P'QR are congruent, $PQ = P'Q$, so that $|p| = |q|$. Therefore, the image formed of an object placed in front of a flat mirror is as far behind the mirror as the object is in front of the mirror.

The geometry in Figure.2 also reveals that the object height h equals the image height h' .

- Let us define lateral magnification M of an image as follows:

$$M = \frac{\text{image height}}{\text{object height}} = \frac{h'}{h}$$

This general definition of the lateral magnification for an image from any type of mirror is also valid for images formed by lenses.

- For a flat mirror, $M = +1$ for any image because $h' = h$. The positive value of the magnification signifies that the image is upright. (By upright we mean that if the object arrow points upward as in Active Figure.2, so does the image arrow.)
- A flat mirror produces an image that has an apparent left-right reversal. You can see this reversal by standing in front of a mirror and raising your right hand. The image you see raises its left hand. Likewise, your hair appears to be parted on the side opposite your real part, and a mole on your right cheek appears to be on your left cheek.
- This reversal is not actually a left-right reversal. The reversal is actually a **front-back** reversal, caused by the light rays going forward toward the mirror and then reflecting back from it. An interesting exercise is to stand in front of a mirror while holding an overhead transparency in front of you so that you can read the writing on the transparency. You will also be able to read the writing on the image of the transparency.

Example 1 : Multiple Images Formed by Two Mirrors

Two flat mirrors are perpendicular to each other as in Figure.3, and an object is placed at point O . In this situation, multiple images are formed. Locate the positions of these images.

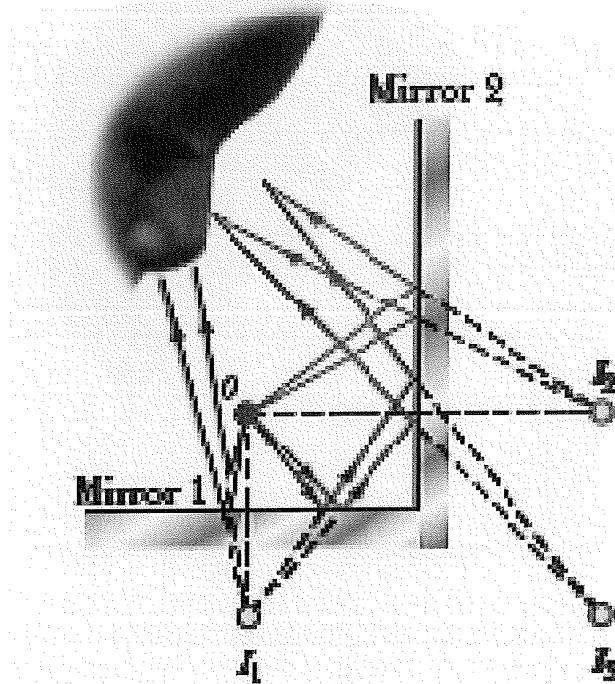


Figure .3 Example.1: When an object is placed in front of two mutually perpendicular mirrors as shown, three images are formed.

SOLUTION

The image of the object is at I_1 in mirror 1 and at I_2 in mirror 2. In addition, a third image is formed at I_3 . This third image is the image of I_1 in mirror 2 or, equivalently, the image of I_2 in mirror 1. That is, the image at I_1 (or I_2) serves as the object for I_3 . To form this image at I_3 , the rays reflect twice after leaving the object at O .

Example 2: The Tilting Rearview Mirror

Most rearview mirrors in cars have a day setting and a night setting. The night setting greatly diminishes the intensity of the image so that

lights from trailing vehicles do not temporarily blind the driver. How does such a mirror work?

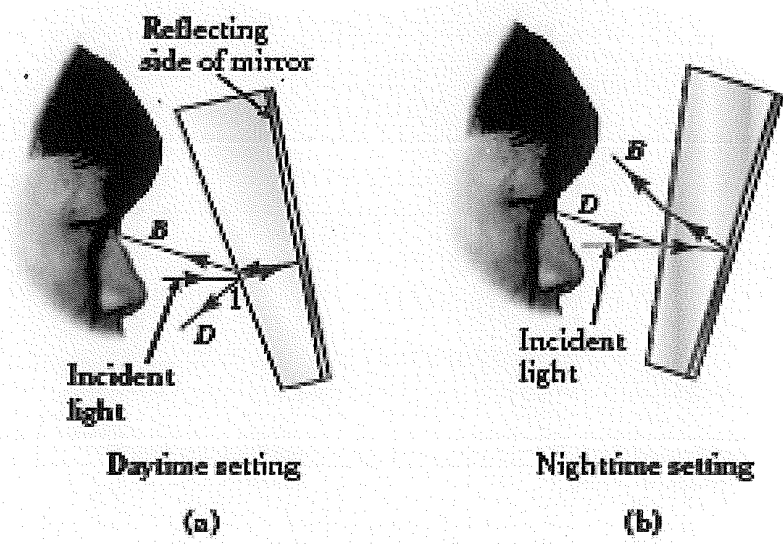


Figure .4 (Example .2) Cross-sectional views of a rearview mirror. (a) With the day setting, (b) With the night setting.

SOLUTION

Figure.4 shows a cross-sectional view of a rearview mirror for each setting. The unit consists of a reflective coating on the back of a wedge of glass. In the day setting (Fig.4a), the light from an object behind the car strikes the glass wedge at point 1. Most of the light enters the wedge, refracting as it crosses the front surface, and reflects from the back surface to return to the front surface, where it is refracted again as it re-enters the air as ray *B* (for *bright*). In addition, a small portion of the light is reflected at the front surface of the glass as indicated by ray *D* (for *dim*).

This dim reflected light is responsible for the image observed when the mirror is in the night setting (Fig.4b). In that case, the wedge is rotated so that the path followed by the bright light (ray *B*) does not lead to the eye. Instead, the dim light reflected from the front surface of the wedge

travels to the eye, and the brightness of trailing headlights does not become a hazard.

2. Images Formed by Spherical Mirrors

In the preceding section, we considered images formed from flat mirrors. Now we study images formed by curved mirrors. Although a variety of curvatures are possible, we will restrict our investigation to spherical mirrors. As its name implies, a spherical mirror has the shape of a section of a sphere.

2.1. Concave Mirrors

We first consider reflection of light from the inner, concave surface of a spherical mirror as shown in Figure 5. This type of reflecting surface is called a concave mirror.

- Figure 5a shows that the mirror has a radius of curvature R , and its center of curvature is point C . Point V is the center of the spherical section, and a line through C and V is called the principal axis of the mirror.

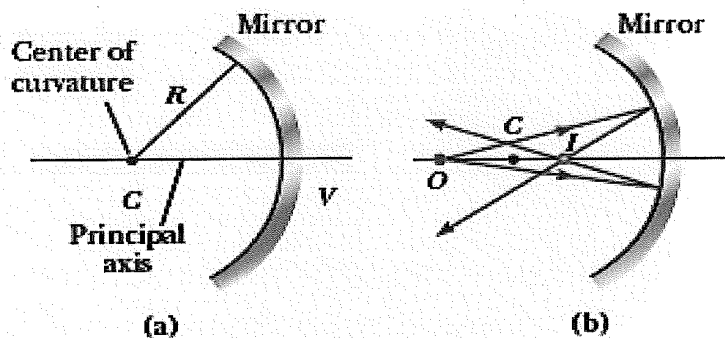


Figure 5 (a) A concave mirror of radius R . The center of curvature C is located on the principal axis. (b) A point object placed at O in front of a concave spherical mirror of radius R .

- Figure 5a shows a cross section of a spherical mirror, with its surface represented by the solid curved line.

- Now consider a point source of light placed at point O in Figure 5b, where O is any point on the principal axis to the left of C . Two diverging light rays that originate at O are shown.
- After reflecting from the mirror, these rays converge and cross at the image point I . They then continue to diverge from I as if an object were there. As a result, the image at point I is real.
- In this section, we shall consider only rays that diverge from the object and make a small angle with the principal axis. Such rays are called **paraxial rays**. All paraxial rays reflect through the image point as shown in Figure 5b.
- Rays that are far from the principal axis such as those shown in Figure 6 converge to other points on the principal axis, producing a blurred image. This effect, called spherical aberration, is present to some extent for any spherical mirror.

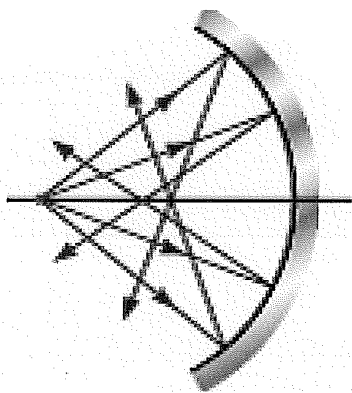


Figure 6 Rays diverging from the object at large angles from the principal axis reflect from a spherical concave mirror to intersect the principal axis at different points, resulting in a blurred image. This condition is called *spherical aberration*.

- If the object distance p and radius of curvature R are known, we can use Figure 7 to calculate the image distance q . By convention, these distances are measured from point V . Figure 7 shows two rays leaving the tip of the object.

- One of these rays passes through the center of curvature C of the mirror, hitting the mirror perpendicular to the mirror surface and reflecting back on itself. The second ray strikes the mirror at its center (point V) and reflects as shown, obeying the law of reflection.
- The image of the tip of the arrow is located at the point where these two rays intersect. From the large, gold right triangle in Figure 7, we see that $\tan \theta = h/p$, and from the blue right triangle, we see that $\tan \theta = -h'/q$. The negative sign is introduced because the image is inverted, so h' is taken to be negative. Therefore, from Equation(1) and these results, we find that the magnification of the image is:

$$M = \frac{h'}{h} = -\frac{q}{p} \quad (2)$$

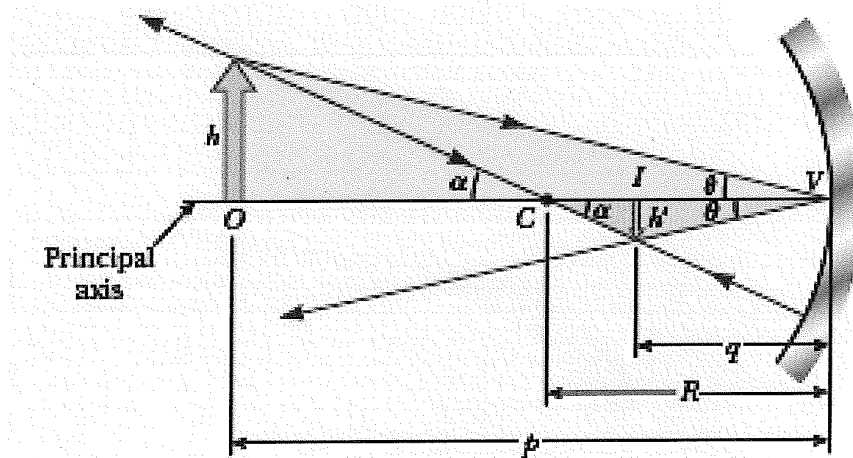


Figure 7 The image formed by a spherical concave mirror when the object O lies outside the center of curvature C .

Also notice from Figure 7 that:

$$\tan \alpha = \frac{-h'}{R - q} \quad \text{and} \quad \tan \alpha = \frac{h}{p - R}$$

from which it follows that

$$\frac{h'}{h} = -\frac{R - q}{p - R} \quad (3)$$

Comparing Equations (2) and (3) gives:

$$\frac{R - q}{p - R} = \frac{q}{p}$$

Simple algebra reduces this expression to:

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} \quad (4)$$

which is called the **mirror equation**.

We present a modified version of this equation shortly.

If the object is very far from the mirror—that is, if p is so much greater than R that p can be said to approach infinity—then $1/p \approx 0$, and Equation.4 shows that $q = R/2$. That is, when the object is very far from the mirror, the image point is halfway between the center of curvature and the center point on the mirror as shown in Figure8a. The incoming rays from the object are essentially parallel in this figure because the source is assumed to be very far from the mirror. The image point in this special case is called the focal point F, and the image distance the focal length f , where

$$f = \frac{R}{2} \quad (5)$$

Because the focal length is a parameter particular to a given mirror, it can be used to compare one mirror with another. Combining

Equations (4) and (5), the mirror equation can be expressed in terms of the focal length:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad (6)$$

Notice that the focal length of a mirror depends only on the curvature of the mirror and not on the material from which the mirror is made because the formation of the image results from rays reflected from the surface of the material. The situation is different for lenses; in that case, the light actually passes through the material and the focal length depends on the type of material from which the lens is made.

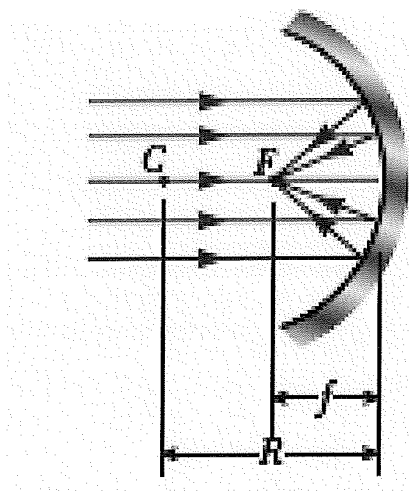


Figure 8 Light rays from a distant object ($p \rightarrow \infty$)

2.2. Convex Mirrors

Figure 9 shows the formation of an image by a convex mirror, that is, one silvered so that light is reflected from the outer, convex surface. It is sometimes called a diverging mirror because the rays from any point on an object diverge after reflection as though they were coming from some point behind the mirror.

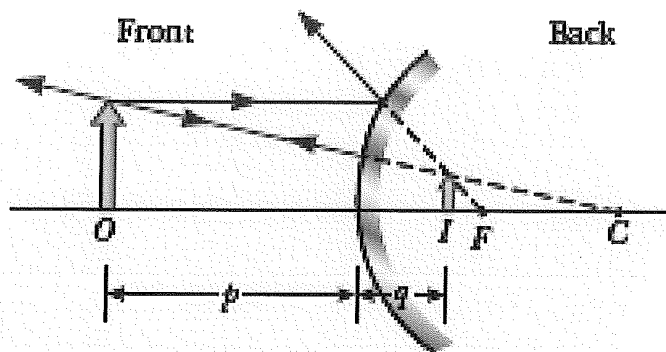


Figure 9 Formation of an image by a spherical convex mirror. The image formed by the object is virtual and upright.

- The image in Figure 9 is virtual because the reflected rays only appear to originate at the image point as indicated by the dashed lines.
- Furthermore, the image is always upright and smaller than the object. This type of mirror is often used in stores to foil shoplifters.
- A single mirror can be used to survey a large field of view because it forms a smaller image of the interior of the store.

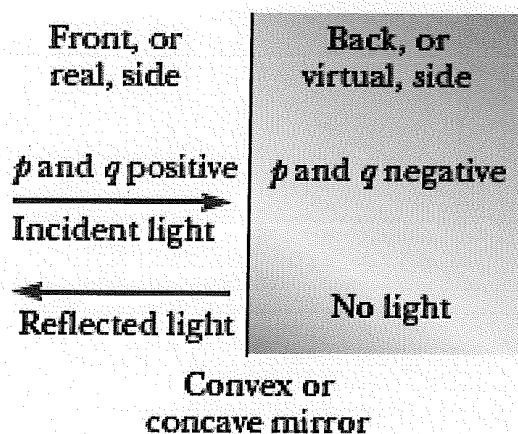


Figure 10 Signs of p and q for convex and concave mirrors.

We do not derive any equations for convex spherical mirrors because Equations (2), (4), and (6) can be used for either concave or convex mirrors if we adhere to the following procedure. We will refer to the

region in which light rays originate and move toward the mirror as the front side of the mirror and the other side as the back side. For example, in Figures 7 and 9, the side to the left of the mirrors is the front side and the side to the right of the mirrors is the back side. Figure 10 states the sign conventions for object and image distances, and Table.1 summarizes the sign conventions for all quantities. One entry in the table, a virtual object, is formally introduced in Section .4.

Ray Diagrams for Mirrors

The positions and sizes of images formed by mirrors can be conveniently determined with ray diagrams. These pictorial representations reveal the nature of the image and can be used to check results calculated from the mathematical representation using the mirror and magnification equations. To draw a ray diagram, you must know the position of the object and the locations of the mirror's focal point and center of curvature. You then draw three rays to locate the image as shown by the examples in Active Figure 11.

Table 1

Sign Conventions for Mirrors		
Quantity	Positive When ...	Negative When ...
Object location (p)	object is in front of mirror (real object).	object is in back of mirror (virtual object).
Image location (q)	image is in front of mirror (real image).	image is in back of mirror (virtual image).
Image height (h')	image is upright.	image is inverted.
Focal length (f) and radius (R)	mirror is concave.	mirror is convex.
Magnification (M)	image is upright.	image is inverted.

These rays all start from the same object point and are drawn as follows. You may choose any point on the object; here, let's choose the

top of the object for simplicity. For concave mirrors (see Active Figs.11a and 11b), draw the following three rays:

- ❖ *Ray 1 is drawn from the top of the object parallel to the principal axis and is reflected through the focal point F .*
- ❖ *Ray 2 is drawn from the top of the object through the focal point (or as if coming from the focal point if $p < f$ and is reflected parallel to the principal axis.*
- ❖ *Ray 3 is drawn from the top of the object through the center of curvature C and is reflected back on itself.*

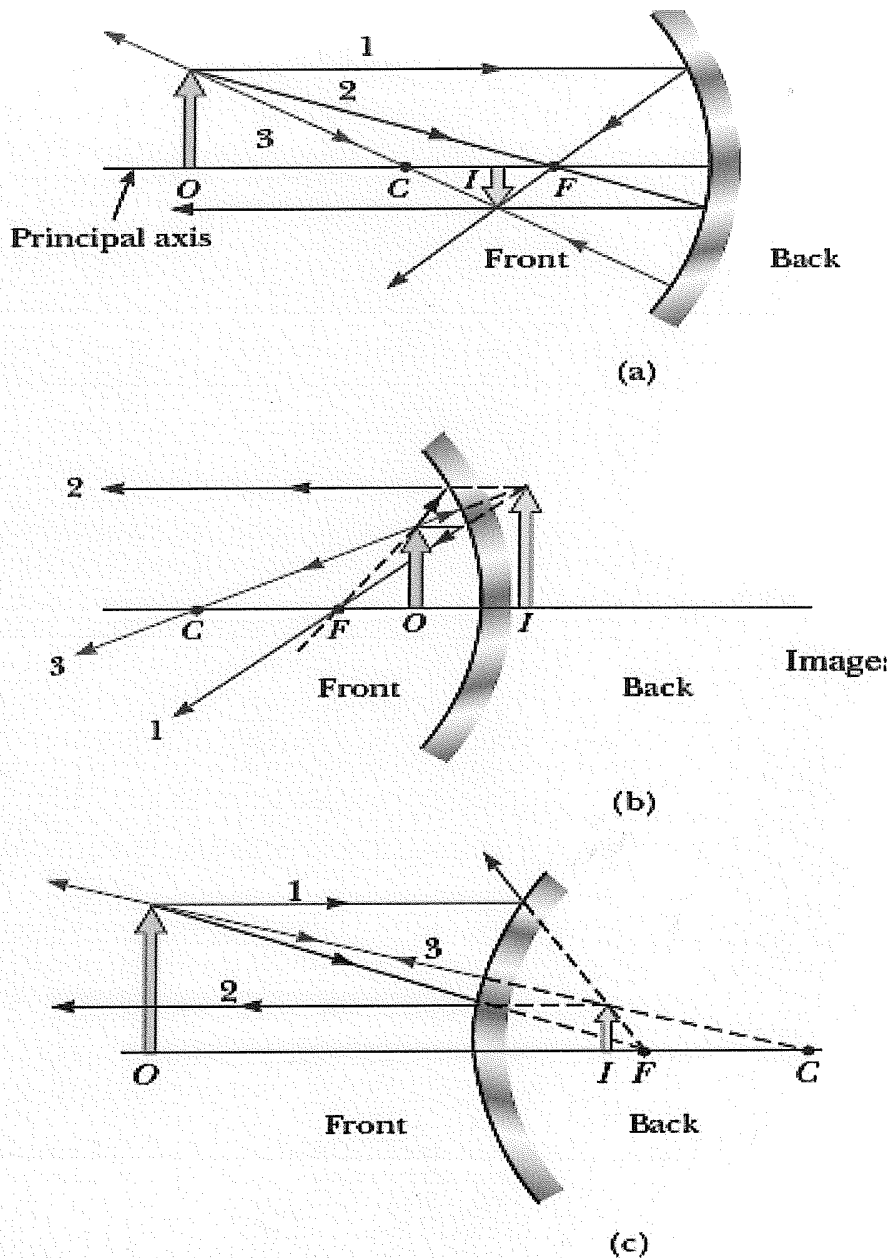


FIGURE .11 Ray diagrams for spherical mirrors along with corresponding photographs of the images of candles. (a) the object is located so that the center of curvature lies between the object and a concave mirror surface. (b) the object is located between the focal point and a concave mirror surface. (c) the object is in front of a convex mirror

The intersection of any two of these rays locates the image. The third ray serves as a check of the construction. The image point obtained in this fashion must always agree with the value of q calculated from the

mirror equation. With concave mirrors, notice what happens as the object is moved closer to the mirror.

- The real, inverted image in Figure 11a moves to the left and becomes larger as the object approaches the focal point.
- When the object is at the focal point, the image is infinitely far to the left.
- When the object lies between the focal point and the mirror surface as shown in Figure.11b, however, the image is to the right, behind the object, and virtual, upright, and enlarged.
- This latter situation applies when you use a shaving mirror or a makeup mirror, both of which are concave. Your face is closer to the mirror than the focal point, and you see an upright, enlarged image of your face

For convex mirrors (see Fig.11c), draw the following three rays:

- ❖ *Ray 1 is drawn from the top of the object parallel to the principal axis and is reflected away from the focal point F.*
- ❖ *Ray 2 is drawn from the top of the object toward the focal point on the back side of the mirror and is reflected parallel to the principal axis.*
- ❖ *Ray 3 is drawn from the top of the object toward the center of curvature C on the back side of the mirror and is reflected back on itself.*

In a convex mirror, the image of an object is always virtual, upright, and reduced in size as shown in Figure.11c. In this case, as the object distance decreases, the virtual image increases in size and moves away from the focal point toward the mirror as the object approaches the

mirror. You should construct other diagrams to verify how image position varies with object position.

Quick Quiz.2 You wish to start a fire by reflecting sunlight from a mirror onto some paper under a pile of wood. Which would be the best choice for the type of mirror? (a) flat (b) concave (c) convex

EXAMPLE 2: The Image Formed by a Concave Mirror

A spherical mirror has a focal length of +10.0 cm.

(A) Locate and describe the image for an object distance of 25.0 cm.

SOLUTION

Because the focal length of the mirror is positive, it is a concave mirror (see Table .1). We expect the possibilities of both real and virtual images. Because the object distance in this part of the problem is larger than the focal length, we expect the image to be real. This situation is analogous to that in Figure.11a. Find the image distance by using Equation .6:

$$\begin{aligned}\frac{1}{q} &= \frac{1}{f} - \frac{1}{p} \\ \frac{1}{q} &= \frac{1}{10.0\text{cm}} - \frac{1}{25.0\text{cm}} \\ q &= 16.7\text{ cm} \\ M &= -\frac{q}{p} = -\frac{16.7\text{cm}}{25.0\text{cm}} = -0.668\end{aligned}$$

- The absolute value of M is less than unity, so the image is smaller than the object, and the negative sign for M tells us that the image is inverted.

- Because q is positive, the image is located on the front side of the mirror and is real.
- Look into the bowl of a shiny spoon or stand far away from a shaving mirror to see this image.

(B) Locate and describe the image for an object distance of 10.0 cm.

SOLUTION

Because the object is at the focal point, we expect the image to be infinitely far away. Find the image distance by using Equation.6:

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{q} = \frac{1}{10.0\text{cm}} - \frac{1}{10.0\text{cm}}$$

$$q = \infty$$

- This result means that rays originating from an object positioned at the focal point of a mirror are reflected so that the image is formed at an infinite distance from the mirror; that is, the rays travel parallel to one another after reflection.
- Such as the situation in a flashlight or an automobile headlight, where the bulb filament is placed at the focal point of a reflector, producing a parallel beam of light.

(C) Locate and describe the image for an object distance of 5.00 cm.

SOLUTION

Find the image distance by using Equation 6:

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{q} = \frac{1}{10.0\text{cm}} - \frac{1}{5.00\text{cm}}$$

$$q = -10.0 \text{ cm}$$

$$M = -\frac{q}{p} = -\left(\frac{-10.0 \text{ cm}}{5.00 \text{ cm}}\right) = +2.00$$

- The image is twice as large as the object, and the positive sign for M indicates that the image is upright (see Fig. 11b).
- The negative value of the image distance tells us that the image is virtual, as expected.
- Put your face close to a shaving mirror to see this type of image.

What If? Suppose you set up the candle and mirror apparatus illustrated in Fig. 11a and described here in part (A). While adjusting the apparatus, you accidentally bump the candle and it begins to slide toward the mirror at velocity v_p . How fast does the image of the candle move?

SOLUTION

Solve the mirror equation, Equation 6, for q :

$$q = \frac{fp}{p-f}$$

$$(1) \quad v_q = \frac{dq}{dt} = \frac{d}{dt} \left(\frac{fp}{p-f} \right) = -\frac{f^2}{(p-f)^2} \frac{dp}{dt} = -\frac{f^2 v_p}{(p-f)^2}$$

$$v_q = -\frac{(10.0 \text{ cm})^2 v_p}{(25.0 \text{ cm} - 10.0 \text{ cm})^2} = -0.444 v_p$$

Therefore, the speed of the image is less than that of the object in this case.

- We can see two interesting behaviors of the function for v_q in Equation (1).

- 1- The velocity is negative regardless of the value of p or f . Therefore, if the object moves toward the mirror, the image moves toward the left in Fig.11 without regard for the side of the focal point at which the object is located or whether the mirror is concave or convex.
- 2- In the limit of $p \rightarrow 0$, the velocity v_q approaches $-v_p$. As the object moves very close to the mirror, the mirror looks like a plane mirror, the image is as far behind the mirror as the object is in front, and both the object and the image move with the same speed.

EXAMPLE 4: The Image Formed by a Convex Mirror

An automobile rearview mirror as shown in Figure 12 shows an image of a truck located 10.0 m from the mirror. The focal length of the mirror is -0.60 m.



Figure 12 (Example .4) An approaching truck is seen in a convex mirror on the right side of an automobile.

(A)-Find the position of the image of the truck.

SOLUTION

This situation is depicted in Fig. 11c

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$
$$\frac{1}{q} = \frac{1}{-0.60 \text{ m}} - \frac{1}{10.0 \text{ m}}$$
$$q = -0.57 \text{ m}$$

Because the mirror is convex, we expect it to form an upright, reduced, virtual image for any object position.

(B)- Find the magnification of the image.

SOLUTION

Use Equation 2:

$$M = -\frac{q}{p} = -\left(\frac{-0.57 \text{ m}}{10.0 \text{ m}}\right) = +0.057$$

The negative value of q in part (A) indicates that the image is virtual, or behind the mirror, as shown in Fig.11c. The magnification in part (B) indicates that the image is much smaller than the truck and is upright because M is positive. Because of the image's small size, these mirrors carry the inscription, "Objects in this mirror are closer than they appear." Look into your rearview mirror or the back side of a shiny spoon to see an image of this type.

3. Images Formed by Refraction

In this section, we describe how images are formed when light rays follow the wave under refraction model at the boundary between two transparent materials. Consider two transparent media having indices of refraction n_1 and n_2 , where the boundary between the two media is a spherical surface of radius R (Fig.13). We assume the object at O is in the medium for which the index of refraction is n_1 . Let's consider the paraxial rays leaving O . As we shall see, all such rays are refracted at the spherical surface and focus at a single point I , the image point.

Figure 14 shows a single ray leaving point O and refracting to point I . Snell's law of refraction applied to this ray gives

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

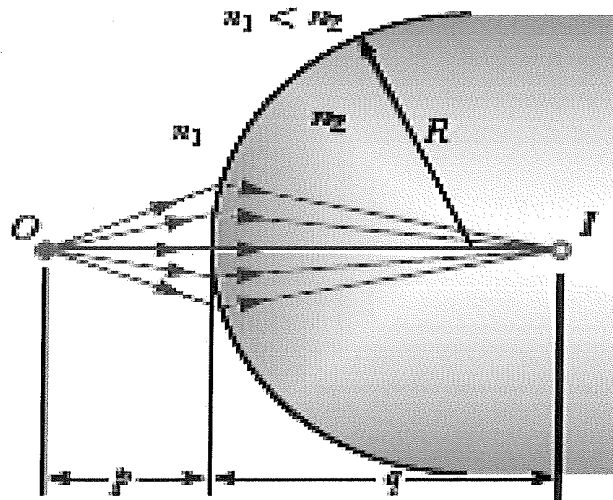


Figure 13 An image formed by refraction at a spherical surface. Rays making small angles with the principal axis diverge from a point object at O and are refracted through the image point I .

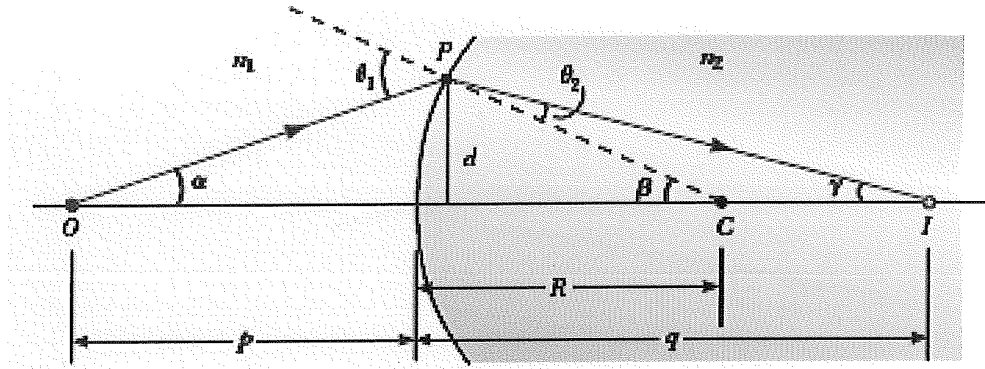


Figure 14 Geometry used to derive Equation 8, assuming that $n_1 < n_2$.

Because θ_1 and θ_2 are assumed to be small, we can use the small-angle approximation $\sin \theta \approx \theta$ (with angles in radians) and write Snell's law as

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

We know that an exterior angle of any triangle equals the sum of the two opposite interior angles, so applying this rule to triangles OPC and PIC in Figure 14 gives:

$$\theta_1 = \alpha + \beta$$

$$\beta = \theta_2 + \gamma$$

Combining all three expressions and eliminating θ_1 and θ_2 gives

$$n_1 \alpha + n_2 \gamma = (n_2 - n_1) \beta \quad (7)$$

Figure 14 shows three right triangles that have a common vertical leg of length d . For paraxial rays, the horizontal legs of these triangles are approximately p for the triangle containing angle α , R for the triangle containing angle β , and q for the triangle containing angle γ . In the

small-angle approximation, $\tan \theta \approx \theta$, so we can write the approximate relationships from these triangles as follows:

$$\tan \alpha \approx \alpha \approx \frac{d}{p} \quad \tan \beta \approx \beta \approx \frac{d}{R} \quad \tan \gamma \approx \gamma \approx \frac{d}{q}$$

Substituting these expressions into Equ.(7) and dividing by d gives:

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \quad (8)$$

- For a fixed object distance p , the image distance q is independent of the angle the ray makes with the axis. This result tells us that all paraxial rays focus at the same point I .
- As with mirrors, we must use a sign convention to apply Equ.(8) to a variety of cases.
- We define the side of the surface in which light rays originate as the front side. The other side is called the back side.
- In contrast with mirrors, where real images are formed in front of the reflecting surface, real images are formed by refraction of light rays to the back of the surface.
- Because of the difference in location of real images, the refraction sign conventions for q and R are opposite the reflection sign conventions. For example, q and R are both positive in Figure 14. The sign conventions for spherical refracting surfaces are summarized in Table.2.
- We derived Equ.(8) from an assumption that $n_1 < n_2$ in Fig.(14). This assumption is not necessary, however. Equ.(8) is valid regardless of which index of refraction is greater

TABLE 2

Sign Conventions for Refracting Surfaces		
Quantity	Positive When ...	Negative When ...
Object location (p)	object is in front of surface (real object).	object is in back of surface (virtual object).
Image location (q)	image is in back of surface (real image).	image is in front of surface (virtual image).
Image height (h')	image is upright.	image is inverted.
Radius (R)	center of curvature is in back of surface.	center of curvature is in front of surface.

Flat Refracting Surfaces

If a refracting surface is flat, then R is infinite and Equation 8 reduces to

$$\frac{n_1}{p} = -\frac{n_2}{q}$$
$$q = -\frac{n_2}{n_1} p$$

From this expression, we see that:

- 1- The sign of q is opposite that of p . Therefore, according to Table 2, the image formed by a flat refracting surface is on the same side of the surface as the object as illustrated in Fig. 15
- 2- For the situation in which the object is in the medium of index n_1 and n_1 is greater than n_2 . In this case, a virtual image is formed between the object and the surface.
- 3- If n_1 is less than n_2 , the rays on the back side diverge from one another at lesser angles than those in Active Fig. 15. As a result, the virtual image is formed to the left of the object.

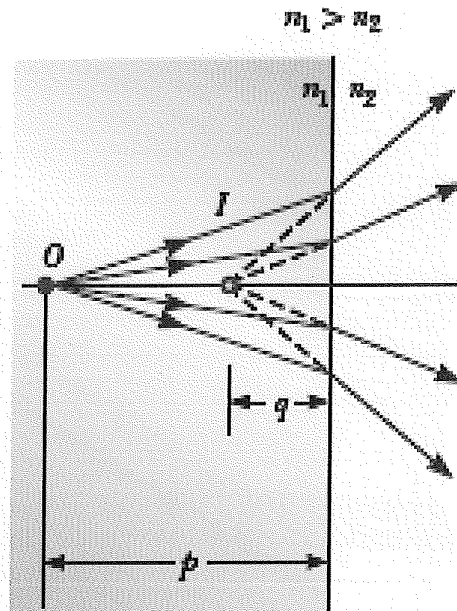


FIGURE 15 The image formed by a flat refracting surface is virtual and on the same side of the surface as the object. All rays are assumed to be paraxial.

Quick Quiz 3 In Figure 13, what happens to the image point I as the object point O is moved to the right from very far away to very close to the refracting surface? (a) It is always to the right of the surface. (b) It is always to the left of the surface. (c) It starts off to the left, and at some position of O , I moves to the right of the surface. (d) It starts off to the right, and at some position of O , I moves to the left of the surface.

Quick Quiz 4 In Active Figure 15, what happens to the image point I as the object point O moves toward the right-hand surface of the material of index of refraction n_1 ? (a) It always remains between O and the surface, arriving at the surface just as O does. (b) It moves toward the surface more slowly than O so that eventually O passes I . (c) It approaches the surface and then moves to the right of the surface

EXAMPLE 5: Let's Go Scuba Diving!

Objects viewed under water with the naked eye appear blurred and out of focus. A scuba diver using a mask, however, has a clear view of underwater objects. Explain how that works, using the information that the indices of refraction of the cornea, water, and air are 1.376, 1.333, and 1.000 29, respectively.

SOLUTION

- Because the cornea and water have almost identical indices of refraction, very little refraction occurs when a person under water views objects with the naked eye. In this case, light rays from an object focus behind the retina, resulting in a blurred image.
- When a mask is used, however, the air space between the eye and the mask surface provides the normal amount of refraction at the eye–air interface; consequently, the light from the object focuses on the retina.

EXAMPLE 6: Gaze into the Crystal Ball

A set of coins is embedded in a spherical plastic paperweight having a radius of 3.0 cm. The index of refraction of the plastic is $n_1 = 1.50$. One coin is located 2.0 cm from the edge of the sphere (Fig.16). Find the position of the image of the coin.

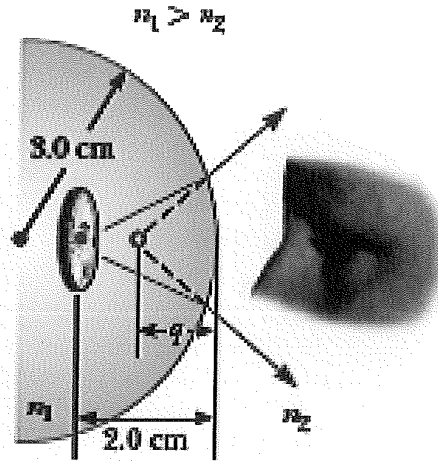


Figure 16 (Example 6) Light rays from a coin embedded in a plastic sphere form a virtual image between the surface of the object and the sphere surface. Because the object is inside the sphere, the front of the refracting surface is the *interior* of the sphere.

SOLUTION

Because $n_1 > n_2$, where $n_2 = 1.00$ is the index of refraction for air, the rays originating from the coin in Figure 16 are refracted away from the normal at the surface and diverge outward.

$$\frac{n_2}{q} = \frac{n_2 - n_1}{R} - \frac{n_1}{p}$$

$$\frac{1}{q} = \frac{1.00 - 1.50}{-3.0\text{cm}} - \frac{1.50}{2.0\text{cm}}$$

$$q = -1.7\text{ cm}$$

The negative sign for q indicates that the image is in front of the surface; in other words, it is in the same medium as the object as shown in Fig.16. Therefore, the image must be virtual. (See Table .2.) The coin appears to be closer to the paperweight surface than it actually is.

EXAMPLE 7: The One That Got Away

A small fish is swimming at a depth d below the surface of a pond (Fig.17).

(A) What is the apparent depth of the fish as viewed from directly overhead?

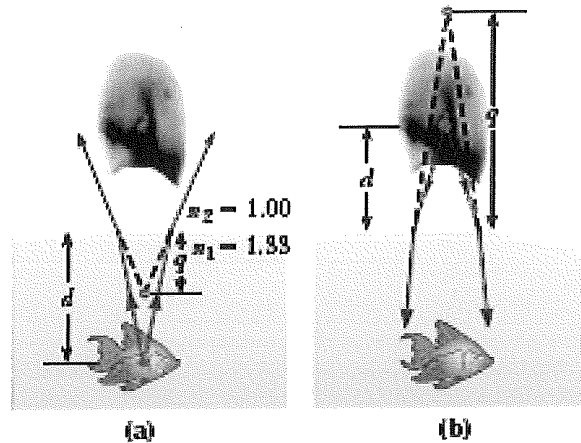


Figure 17 (Example 7) (a) The apparent depth q of the fish is less than the true depth d . All rays are assumed to be paraxial. (b) Your face appears to the fish to be higher above the surface than it is.

SOLUTION

Use the indices of refraction given in Figure 17a in Equ.(9):

$$q = -\frac{n_2}{n_1} p = -\frac{1.00}{1.33} d = -0.752 d$$

Because q is negative, the image is virtual as indicated by the dashed lines in Figure 17a. The apparent depth is approximately three-fourths the actual depth.

(B) If your face is a distance d above the water surface, at what apparent distance above the surface does the fish see your face?

SOLUTION

The light rays from your face are shown in Figure 17b.

- Because the rays refract toward the normal, your face appears higher above the surface than it actually is.
- And because the refracting surface is flat, R is infinite. Hence, we can use Equ.(9) to determine the location of the image with $p=d$.

$$q = -\frac{n_2}{n_1} p = -\frac{1.33}{1.00} d = -1.33 d$$

- The negative sign for q indicates that the image is in the medium from which the light originated, which is the air above the water.

What If? What if you look more carefully at the fish and measure its apparent height from its upper fin to its lower fin? Is the apparent height h' of the fish different from the actual height h ?

Answer Because all points on the fish appear to be fractionally closer to the observer, we expect the height to be smaller. Let the distance d in Fig.17a be measured to the top fin, and let the distance to the bottom fin be $d + h$. Then the images of the top and bottom of the fish are located at

$$q_{top} = -0.752 d$$

$$q_{bottom} = -0.752(d + h)$$

the apparent height h' of the fish is

$$h' = q_{top} - q_{bottom} = -0.752 - [-0.752(d+h)] = 0.752h$$

Hence, the fish appears to be approximately three-fourths its actual height.

4. Thin Lenses

Lenses are commonly used to form images by refraction in optical instruments such as cameras, telescopes, and microscopes. Let's use what we just learned about images formed by refracting surfaces to help locate the image formed by a lens.

- Light passing through a lens experiences refraction at two surfaces. The development we shall follow is based on the notion that the image formed by one refracting surface serves as the object for the second surface. We shall analyze a thick lens first and then let the thickness of the lens be approximately zero.
- Consider a lens having an index of refraction n and two spherical surfaces with radii of curvature R_1 and R_2 as in Fig.18. Notice that R_1 is the radius of curvature of the lens surface the light from the object reaches first and R_2 is the radius of curvature of the other surface of the lens.
- An object is placed at point O at a distance p_1 in front of surface 1. Let's begin with the image formed by surface 1. Using Equ.8 and assuming $n_1 = 1$ because the lens is surrounded by air, we find that the image I_1 formed by surface 1 satisfies the equation

$$\frac{1}{p_1} + \frac{n}{q_1} = \frac{n-1}{R_1} \quad (10)$$

where q_1 is the position of the image formed by surface 1.

- If the image formed by surface 1 is virtual (Fig. 18a), q_1 is negative; it is positive if the image is real (Fig. 18b).
- Now let's apply Equ.8 to surface 2, taking $n_1 = n$ and $n_2 = 1$. (We make this switch in index because the light rays approaching

surface 2 are in the material of the lens, and this material has index n .)

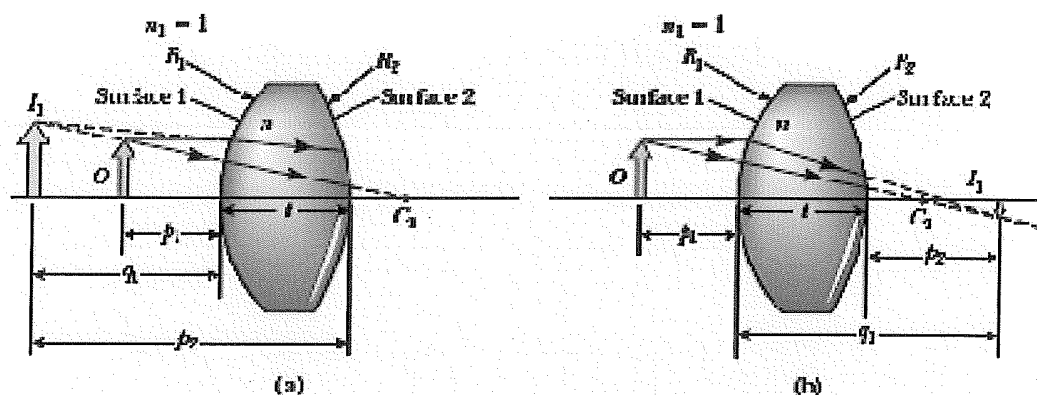


Figure 18 To locate the image formed by a lens, we use the virtual image at I_1 formed by surface 1 as the object for the image formed by surface 2. The point C_1 is the center of curvature of surface 1. (a) The image due to surface 1 is virtual, so I_1 is to the left of the surface. (b) The image due to surface 1 is real, so I_1 is to the right of the surface.

Taking p_2 as the object distance for surface 2 and q_2 as the image distance gives

$$\frac{n}{p_2} + \frac{1}{q_2} = \frac{1-n}{R_2} \quad (11)$$

- We now introduce mathematically that the image formed by the first surface acts as the object for the second surface. If the image from surface 1 is virtual as in Fig.18a, we see that p_2 , measured from surface 2, is related to q_1 as $p_2 = -q_1 + t$, where t is the thickness of the lens. Because q_1 is negative, p_2 is a positive number.
- Fig.18b shows the case of the image from surface 1 being real. In this situation, q_1 is positive and $p_2 = -q_1 + t$, where the image from surface 1 acts as a virtual object, so p_2 is negative. Regardless of

the type of image from surface 1, the same equation describes the location of the object for surface 2 based on our sign convention.

- For a thin lens (one whose thickness is small compared with the radii of curvature), we can neglect t . In this approximation, $p_2 = q_1$ for either type of image from surface 1. Hence, Equ.(11) becomes

$$-\frac{n}{q_1} + \frac{1}{q_2} = \frac{1-n}{R_2}$$

$$\frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

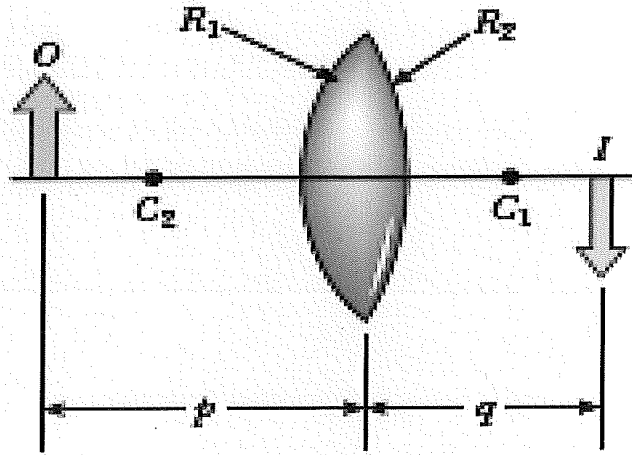


Figure 19 Simplified geometry for a thin lens.

$$\frac{1}{p} + \frac{1}{q} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad (14)$$

$$\boxed{\frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)} \quad (15)$$

- This relationship (Equ.15) is called the lens-makers' equation because it can be used to determine the values of R_1 and R_2 needed for a given index of refraction and a desired focal length f .
- Conversely, if the index of refraction and the radii of curvature of a lens are given, this equation can be used to find the focal length.
- If the lens is immersed in something other than air, this same equation can be used, with n interpreted as the ratio of the index of refraction of the lens material to that of the surrounding fluid

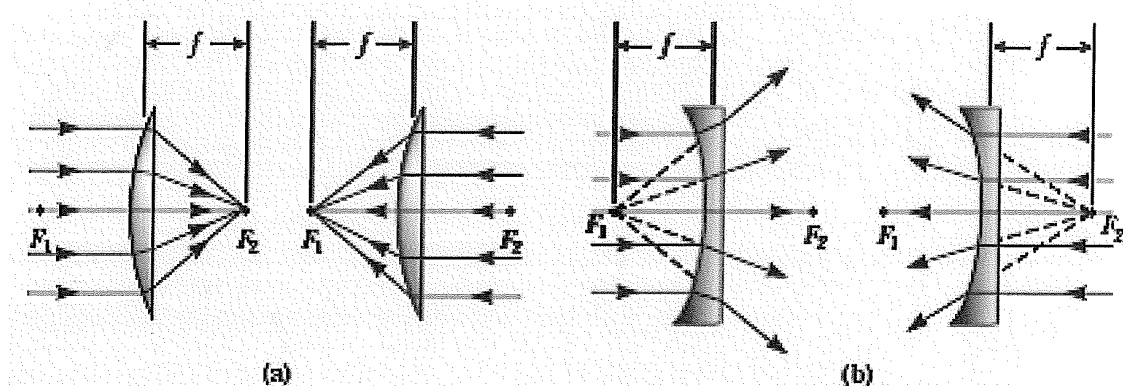


Figure 20 Parallel light rays pass through (a) a converging lens and (b) a diverging lens. The focal length is the same for light rays passing through a given lens in either direction. Both focal points F_1 and F_2 are the same distance from the lens.

Using Equ.15, we can write Equ.14 in a form identical to Equ.(6) for mirrors:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad (16)$$

This equation, called the thin lens equation, can be used to relate the image distance and object distance for a thin lens.

- Because light can travel in either direction through a lens, each lens has two focal points, one for light rays passing through in one direction and one for rays passing through in the other direction.
- These two focal points are illustrated in Fig.20 for a plano-convex lens (a converging lens) and a plano-concave lens.
- Fig.21 is useful for obtaining the signs of p and q , and Table 3 gives the sign conventions for thin lenses. These sign conventions are the same as those for refracting surfaces (see Table 2).
- Various lens shapes are shown in Fig.22. Notice that a converging lens is thicker at the center than at the edge, whereas a diverging lens is thinner at the center than at the edge.

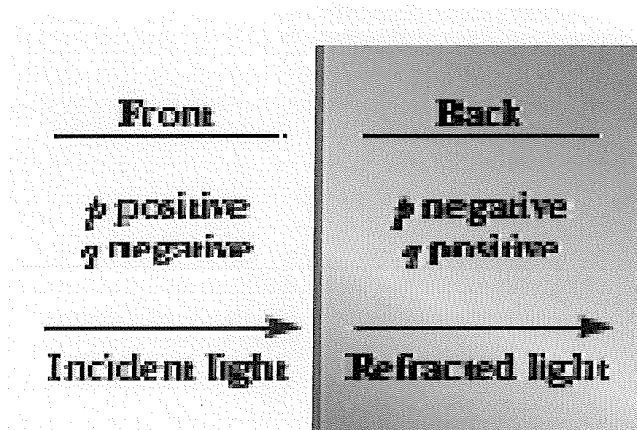


Figure 21 A diagram for obtaining the signs of p and q for a thin lens. (This diagram also applies to a refracting surface.)

Magnification of Images

Consider a thin lens through which light rays from an object pass. As with mirrors (Eq.2), a geometric construction shows that the lateral magnification of the image is

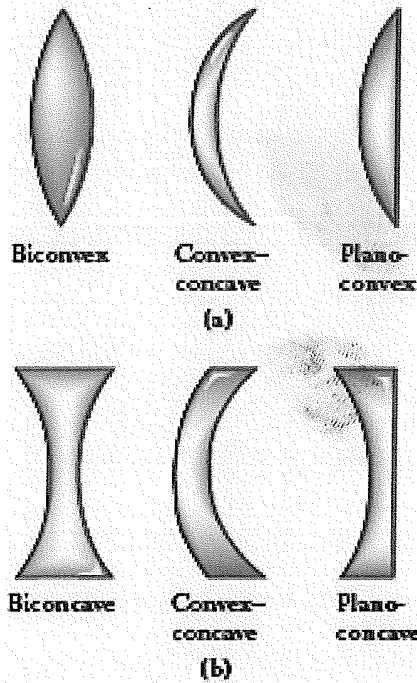


Figure 22 Various lens shapes. (a) Converging lenses have a positive focal length and are thickest at the middle. (b) Diverging lenses have a negative focal length and are thickest at the edges.

$$M = \frac{h'}{h} = -\frac{q}{p}$$

From this expression, it follows that when M is positive, the image is upright and on the same side of the lens as the object. When M is negative, the image is inverted and on the side of the lens opposite the object.

TABLE 3

Sign Conventions for Thin Lenses		
Quantity	Positive When ...	Negative When ...
Object location (p)	object is in front of lens (real object).	object is in back of lens (virtual object).
Image location (q)	image is in back of lens (real image).	image is in front of lens (virtual image).
Image height (h')	image is upright.	image is inverted.
R_1 and R_2	center of curvature is in back of lens.	center of curvature is in front of lens.
Focal length (f)	a converging lens.	a diverging lens.

Ray Diagrams for Thin Lenses

Ray diagrams are convenient for locating the images formed by thin lenses or systems of lenses. They also help clarify our sign conventions. Fig.23 shows such diagrams for three single-lens situations. To locate the image of a converging lens (Fig.23a and b), the following three rays are drawn from the top of the object:

- ❖ Ray 1 is drawn parallel to the principal axis. After being refracted by the lens, this ray passes through the focal point on the back side of the lens.
- ❖ Ray 2 is drawn through the center of the lens and continues in a straight line.
- ❖ Ray 3 is drawn through the focal point on the front side of the lens and emerges from the lens parallel to the principal axis.

To locate the image of a diverging lens (Fig.23c), the following three rays are drawn from the top of the object:

- ❖ 1 is drawn parallel to the principal axis. After being refracted by the lens, this ray emerges directed away from the focal point on the front side of the lens.

- ❖ Ray 2 is drawn through the center of the lens and continues in a straight line.
- ❖ Ray 3 is drawn in the direction toward the focal point on the back side of the lens and emerges from the lens parallel to the principal axis.

For the converging lens

- In Fig.23a, where the object is to the left of the focal point ($p > f$), the image is real and inverted.
- When the object is between the focal point and the lens ($p < f$) as in Fig.23b, the image is virtual and upright. In that case, the lens acts as a magnifying glass
- For a diverging lens Fig.23c, the image is always virtual and upright, regardless of where the object is placed.
- These geometric constructions are reasonably accurate only if the distance between the rays and the principal axis is much less than the radii of the lens surfaces.

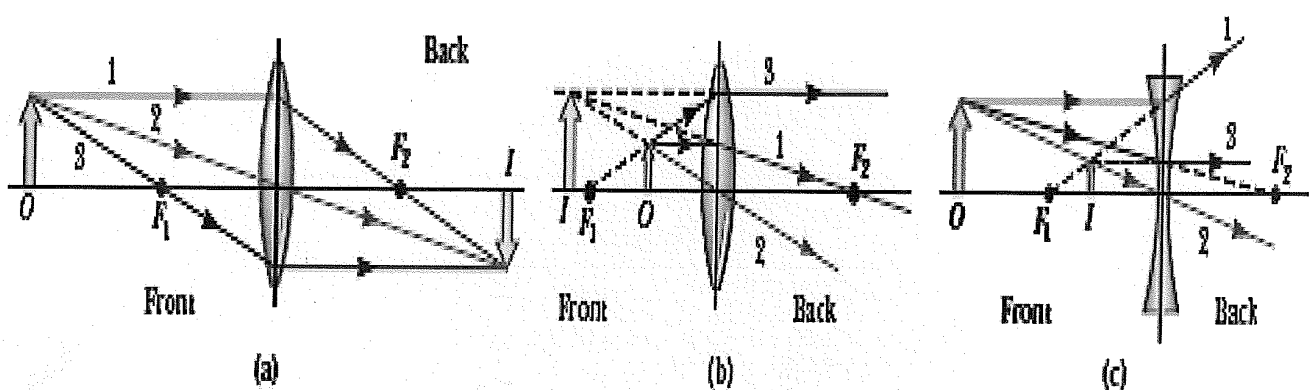


FIGURE .23 Ray diagrams for locating the image formed by a thin lens. (a) When the object is in front of and outside the focal point of a converging lens, the image is real, inverted, and on the back side of the lens. (b) When the object is between the focal point and a converging lens, the image is virtual, upright, larger than the object, and on the front side of the lens. (c) When an object is anywhere in front of a diverging lens, the image is virtual, upright, smaller than the object, and on the front side of the lens.

Quick Quiz .5 What is the focal length of a pane of window glass?
 (a) zero (b) infinity (c) the thickness of the glass (d) impossible to determine

EXAMPLE 8 : Images Formed by a Converging Lens

A converging lens has a focal length of 10.0 cm.

(A) An object is placed 30.0 cm from the lens. Construct a ray diagram, find the image distance, and describe the image.

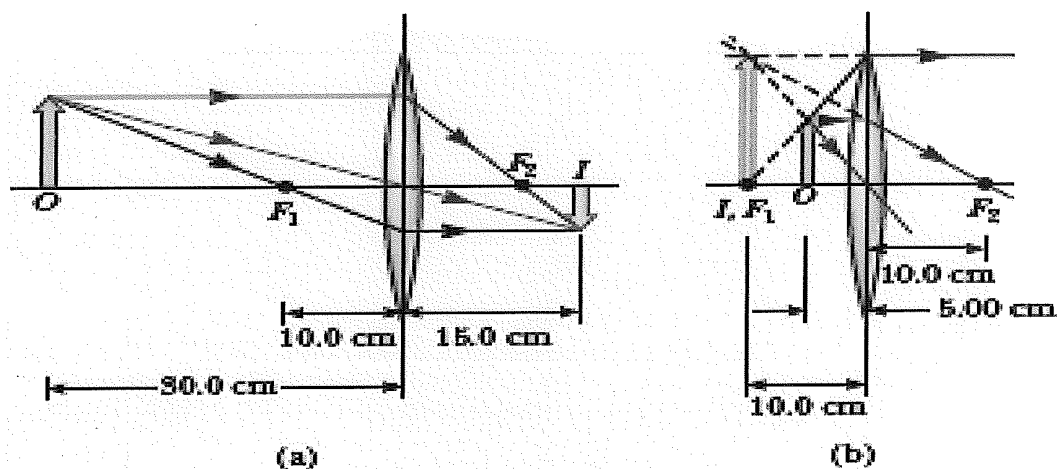


Figure .24 (Example.8) An image is formed by a converging lens. (a) The object is farther from the lens than the focal point. (b) The object is closer to the lens than the focal point.

SOLUTION

Find the image distance by using Equ.16:

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{q} = \frac{1}{10.0 \text{ cm}} - \frac{1}{30.0 \text{ cm}}$$

$$q = +15.0 \text{ cm}$$

$$M = -\frac{q}{p} = -\frac{15.0 \text{ cm}}{30.0 \text{ cm}} = -0.500$$

The positive sign for the image distance tells us that the image is indeed real and on the back side of the lens. The magnification of the image tells us that the image is reduced in height by one half, and the negative sign for M tells us that the image is inverted.

(B) An object is placed 10.0 cm from the lens. Find the image distance and describe the image.

SOLUTION

Because the object is at the focal point, we expect the image to be infinitely far away. Find the image distance by using Equ.16:

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{q} = \frac{1}{10.0 \text{ cm}} - \frac{1}{10.0 \text{ cm}}$$

$$q = \infty$$

(C) An object is placed 5.00 cm from the lens. Construct a ray diagram, find the image distance, and describe the image.

SOLUTION

Because the object distance is smaller than the focal length, we expect the image to be virtual. The ray diagram for this situation is shown in Figure .24b.

Find the image distance by using Equ.16:

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{q} = \frac{1}{10.0 \text{ cm}} - \frac{1}{5.00 \text{ cm}}$$

$$q = -10.0 \text{ cm}$$

$$M = -\frac{q}{p} = -\left(\frac{-10.0 \text{ cm}}{5.00 \text{ cm}}\right) = +2.00$$

The negative image distance tells us that the image is virtual and formed on the side of the lens from which the light is incident, the front side. The image is enlarged, and the positive sign for M tells us that the image is upright.

What If? What if the object moves right up to the lens surface, so that $p \rightarrow 0$? Where is the image?

Answer In this case, because $p \ll R$, where R is either of the radii of the surfaces of the lens, the curvature of the lens can be ignored. The lens should appear to have the same effect as a flat piece of material, which suggests that the image is just on the front side of the lens, at $q=0$. This conclusion can be verified mathematically by rearranging the thin lens equation:

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

If we let $p \rightarrow 0$, the second term on the right becomes very large compared with the first and we can neglect $1/f$.

The equation becomes

$$\frac{1}{q} = -\frac{1}{p} \rightarrow q = -p = 0$$

Therefore, q is on the front side of the lens (because it has the opposite sign as p and right at the lens surface).

EXAMPLE 9: Images Formed by a Diverging Lens

A diverging lens has a focal length of 10.0 cm.

(A) An object is placed 30.0 cm from the lens. Construct a ray diagram, find the image distance, and describe the image.

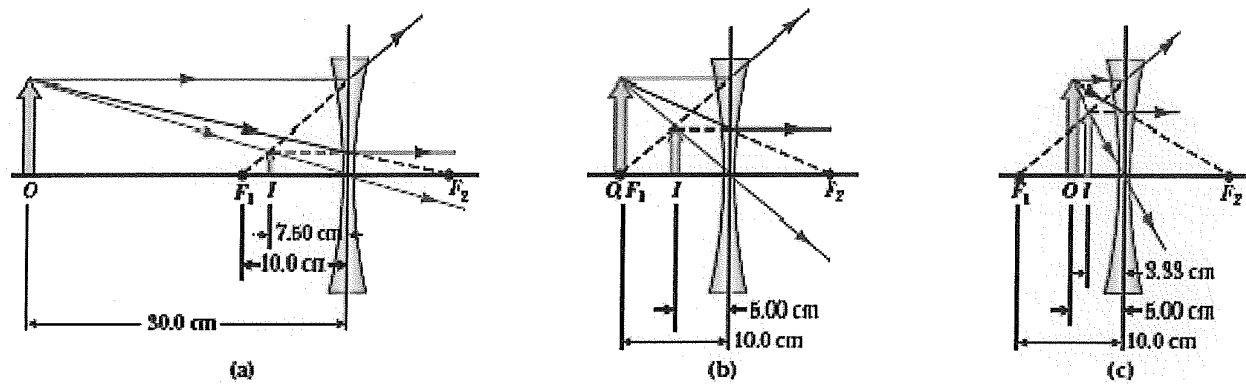


Figure 25 (Example.9) An image is formed by a diverging lens. (a) The object is farther from the lens than the focal point. (b) The object is at the focal point. (c) The object is closer to the lens than the focal point.

SOLUTION

Because the lens is diverging, the focal length is negative (see Table.3). The ray diagram for this situation is shown in Fig.25a. Because the lens is diverging, we expect it to form an upright, reduced, virtual image for any object position.

Find the image distance by using Equ.16:

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{q} = \frac{1}{-10.0\text{ cm}} - \frac{1}{30.0\text{ cm}}$$

$$q = -7.50\text{ cm}$$

$$M = -\frac{q}{p} = -\left(\frac{-7.50\text{ cm}}{30.0\text{ cm}}\right) = +0.250$$

This result confirms that the image is virtual, smaller than the object, and upright. Look through the diverging lens in a door peephole to see this type of image.

(B) An object is placed 10.0 cm from the lens. Construct a ray diagram, find the image distance, and describe the image.

SOLUTION

The ray diagram for this situation is shown in Fig. 25b.

Find the image distance by using Equ. 16:

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{q} = \frac{1}{-10.0 \text{ cm}} - \frac{1}{10.0 \text{ cm}}$$

$$q = -5.00 \text{ cm}$$

$$M = -\frac{q}{p} = -\left(\frac{-5.00 \text{ cm}}{10.0 \text{ cm}}\right) = +0.500$$

Notice the difference between this situation and that for a converging lens. For a diverging lens, an object at the focal point does not produce an image infinitely far away.

(C) An object is placed 5.00 cm from the lens. Construct a ray diagram, find the image distance, and describe the image.

SOLUTION

The ray diagram for this situation is shown in Fig. 25c.

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{q} = \frac{1}{-10.0 \text{ cm}} - \frac{1}{5.00 \text{ cm}}$$

$$q = -3.33 \text{ cm}$$

$$M = -\left(\frac{-3.33 \text{ cm}}{5.00 \text{ cm}}\right) = +0.667$$

For all three object positions, the image position is negative and the magnification is a positive number smaller than 1, which confirms that the image is virtual, smaller than the object, and upright

Combination of Thin Lenses

If two thin lenses are used to form an image, the system can be treated in the following manner:

- 1- the image formed by the first lens is located as if the second lens were not present.
- 2- Then a ray diagram is drawn for the second lens, with the image formed by the first lens now serving as the object for the second lens.
- 3- The second image formed is the final image of the system.

If the image formed by the first lens lies on the back side of the second lens, that image is treated as a virtual object for the second lens (that is, in the thin lens equation, p is negative).

The same procedure can be extended to a system of three or more lenses. Because the magnification due to the second lens is performed on the magnified image due to the first lens, **the overall magnification of**

the image due to the combination of lenses is the product of the individual magnifications:

$$M = M_1 M_2 \quad (18)$$

This equation can be used for combinations of any optical elements such as a lens and a mirror. For more than two optical elements, the magnifications due to all elements are multiplied together.

Let's consider the special case of a system of two lenses of focal lengths f_1 and f_2 in contact with each other. If $p_1 = p$ is the object distance for the combination, application of the thin lens equation (Eq.16) to the first lens gives:

$$\frac{1}{p} + \frac{1}{q_1} = \frac{1}{f_1}$$

where q_1 is the image distance for the first lens. Treating this image as the object for the second lens, we see that the object distance for the second lens must be $p_2 = -q_1$. (The distances are the same because the lenses are in contact and assumed to be infinitesimally thin. The object distance is negative because the object is virtual.)

Therefore, for the second lens,

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2} \rightarrow -\frac{1}{q_1} + \frac{1}{q} = \frac{1}{f_2}$$

where $q = q_2$ is the final image distance from the second lens, which is the image distance for the combination. Adding the equations for the two lenses eliminates q_1 and gives:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f_1} + \frac{1}{f_2}$$

If the combination is replaced with a single lens that forms an image at the same location, its focal length must be related to the individual focal lengths by the expression

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \quad (19)$$

Therefore, **two thin lenses in contact with each other are equivalent to a single thin lens having a focal length given by Eq.19.**

EXAMPLE 10: Where Is the Final Image?

Two thin converging lenses of focal lengths $f_1=10.0$ cm and $f_2= 20.0$ cm are separated by 20.0 cm as illustrated in Figure.30. An object is placed 30.0 cm to the left of lens

1. Find the position and the magnification of the final image.

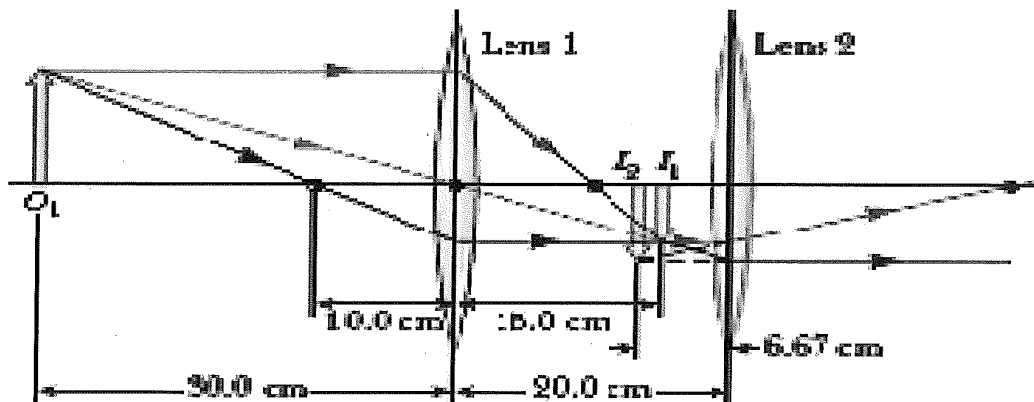


Figure.26 (Example.10) A combination of two converging lenses.

SOLUTION

- Imagine light rays passing through the first lens and forming a real image (because $p > f$) in the absence of a second lens.
- Figure 26 shows these light rays forming the inverted image I_1 .

- Once the light rays converge to the image point, they do not stop. They continue through the image point and interact with the second lens.
- The rays leaving the image point behave in the same way as the rays leaving an object. Therefore, the image of the first lens serves as the object of the second lens.
- Find the location of the image formed by lens 1 from the thin lens equation

$$\frac{1}{q_1} = \frac{1}{f} - \frac{1}{p_1}$$

$$\frac{1}{q_1} = \frac{1}{10.0 \text{ cm}} - \frac{1}{30.0 \text{ cm}}$$

$$q_1 = +15.0 \text{ cm}$$

$$M_1 = -\frac{q_1}{p_1} = -\frac{15.0 \text{ cm}}{30.0 \text{ cm}} = -0.500$$

- The image formed by this lens acts as the object for the second lens. Therefore, the object distance for the second lens is $20.0 \text{ cm} - 15.0 \text{ cm} = 5.00 \text{ cm}$.

$$\frac{1}{q_2} = \frac{1}{20.0 \text{ cm}} - \frac{1}{5.00 \text{ cm}}$$

$$q_2 = -6.67 \text{ cm}$$

$$M_2 = -\frac{q_2}{p_2} = -\frac{(-6.67 \text{ cm})}{5.00 \text{ cm}} = +1.33$$

$$M = M_1 M_2 = (-0.500)(1.33) = -0.667$$

The negative sign on the overall magnification indicates that the final image is inverted with respect to the initial object. Because the absolute value of the magnification is less than 1, the final image is

smaller than the object. Because q_2 is negative, the final image is on the front, or left, side of lens 2. These conclusions are consistent with the ray diagram in Fig. 26.

What If? Suppose you want to create an upright image with this system of two lenses. How must the second lens be moved?

Answer Because the object is farther from the first lens than the focal length of that lens, the first image is inverted. Consequently, the second lens must invert the image once again so that the final image is upright. An inverted image is only formed by a converging lens if the object is outside the focal point. Therefore, the image formed by the first lens must be to the left of the focal point of the second lens in Fig. 26. To make that happen, you must move the second lens at least as far away from the first lens as the sum

$$q_1 + f_2 = 15.0 \text{ cm} + 20.0 \text{ cm} = 35.0 \text{ cm}.$$

5. Lens Aberrations

Our analysis of mirrors and lenses assumes rays make small angles with the principal axis and the lenses are thin. In this simple model, all rays leaving a point source focus at a single point, producing a sharp image. Clearly, that is not always true. When the approximations used in this analysis do not hold, imperfect images are formed.

A precise analysis of image formation requires tracing each ray, using Snell's law at each refracting surface and the law of reflection at each reflecting surface. This procedure shows that the rays from a point object do not focus at a single point, with the result that the

image is blurred. The departures of actual images from the ideal predicted by our simplified model are called aberrations.

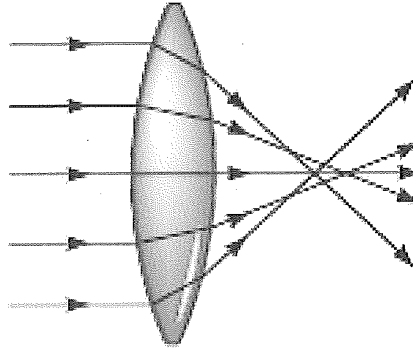


Figure 27 Spherical aberration caused by a converging lens. Does a diverging lens cause spherical aberration?

Spherical Aberration

Spherical aberration occurs because the focal points of rays far from the principal axis of a spherical lens (or mirror) are different from the focal points of rays of the same wavelength passing near the axis. Figure 27 illustrates spherical aberration for parallel rays passing through a converging lens. Rays passing through points near the center of the lens are imaged farther from the lens than rays passing through points near the edges. Figure.6 earlier in the chapter showed a similar situation for a spherical mirror.

Many cameras have an adjustable aperture to control light intensity and reduce spherical aberration. (An aperture is an opening that controls the amount of light passing through the lens.) Sharper images are produced as the aperture size is reduced; with a small aperture, only the central portion of the lens is exposed to the light and therefore a greater percentage of the rays are paraxial. At the

same time, however, less light passes through the lens. To compensate for this lower light intensity, a longer exposure time is used.

In the case of mirrors, spherical aberration can be minimized through the use of a parabolic reflecting surface rather than a spherical surface. Parabolic surfaces are not used often, however, because those with high-quality optics are very expensive to make. Parallel light rays incident on a parabolic surface focus at a common point, regardless of their distance from the principal axis. Parabolic reflecting surfaces are used in many astronomical telescopes to enhance image quality.

Chromatic Aberration

In Chapter 1, we described dispersion, whereby a material's index of refraction varies with wavelength. Because of this phenomenon, violet rays are refracted more than red rays when white light passes through a lens (Fig. 28). The figure shows that the focal length

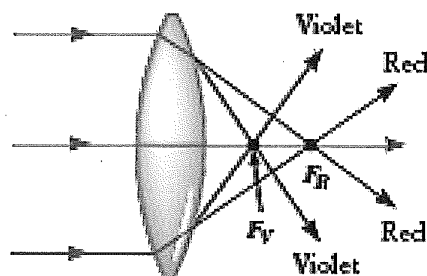


Figure 28 Chromatic aberration caused by a converging lens. Rays of different wavelengths focus at different points.

of a lens is greater for red light than for violet light.

Other wavelengths (not shown in Fig. 28) have focal points intermediate between those of red and violet, which causes a blurred image and is called chromatic aberration.

Chromatic aberration for a diverging lens also results in a shorter focal length for violet light than for red light, but on the front side of the lens. Chromatic aberration can be greatly reduced by combining a converging lens made of one type of glass and a diverging lens made of another type of glass.

6. The Camera

The photographic camera is a simple optical instrument whose essential features are shown in Figure 29. It consists of a light chamber, a converging lens that produces a real image, and a film behind the lens to receive the image.

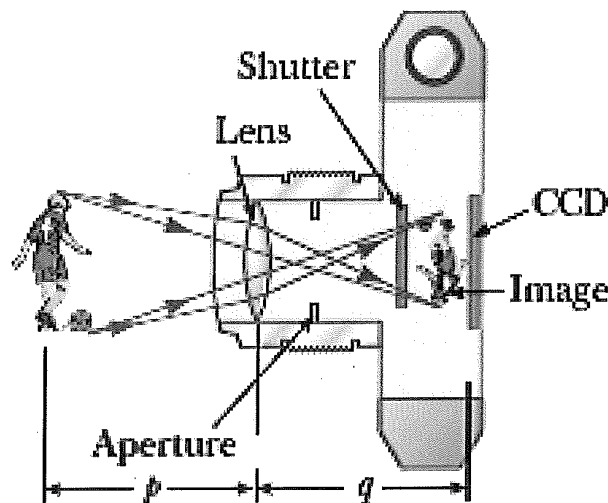


Figure 29 Cross-sectional view of a simple digital camera. The CCD is the light-sensitive component of the camera. In a nondigital camera, the light from the lens falls onto photographic

film. In reality, $p \gg q$.

Digital cameras are similar to film cameras except that the light does not form an image on photographic film. The image in a digital camera is formed on a charge-coupled device (CCD), which digitizes the image, turning it into binary code. The digital information is then stored on a

memory chip for playback on the camera's display screen, or it can be downloaded to a computer. In the discussion that follows, we assume the camera is digital.

A camera is focused by varying the distance between the lens and the CCD. For proper focusing—which is necessary for the formation of sharp images—the lens to- CCD distance depends on the object distance as well as the focal length of the lens.

The shutter, positioned behind the lens, is a mechanical device that is opened for selected time intervals, called exposure times. You can photograph moving objects by using short exposure times or photograph dark scenes (with low light levels) by using long exposure times. If this adjustment were not available, it would be impossible to take stop-action photographs. For example, a rapidly moving vehicle could move enough in the time interval during which the shutter is open to produce a blurred image. Another major cause of blurred images is the movement of the camera while the shutter is open. To prevent such movement, either short exposure times or a tripod should be used, even for stationary objects. Typical shutter speeds (that is, exposure times) are $\frac{1}{30}$ s, $\frac{1}{60}$ s, $\frac{1}{125}$ s, and $\frac{1}{250}$ s. In practice, stationary objects are normally shot with an intermediate shutter speed of $\frac{1}{60}$ s.

The intensity I of the light reaching the CCD is proportional to the area of the lens. Because this area is proportional to the square of the diameter D , it follows that I is also proportional to D^2 . Light intensity is a measure of the rate at which energy is received by the CCD per unit area of the image. Because the area of the image is proportional to q^2 and $q \approx f$ (when $p \gg f$, so p can be approximated as

infinite), we conclude that the intensity is also proportional to $1/f^2$ and therefore that $I \propto D^2/f^2$.

The ratio f/D is called the f -number of a lens:

$$f\text{-number} \equiv \frac{f}{D} \quad (20)$$

Hence, the intensity of light incident on the CCD varies according to the following proportionality:

$$I \propto \frac{1}{(f/D)^2} \propto \frac{1}{(f\text{-number})^2} \quad (36.21)$$

The f -number is often given as a description of the lens's "speed." The lower the f -number, the wider the aperture and the higher the rate at which energy from the light exposes the CCD; therefore, a lens with a low f -number is a "fast" lens. The conventional notation for an f -number is " $f/$ " followed by the actual number. For example, " $f/4$ " means an f -number of 4; it does not mean to divide f by 4!

Extremely fast lenses, which have f -numbers as low as approximately $f/1.2$, are expensive because it is very difficult to keep aberrations acceptably small with light rays passing through a large area of the lens. Camera lens systems (that is, combinations of lenses with adjustable apertures) are often marked with multiple f -numbers, usually $f/2.8$, $f/4$, $f/5.6$, $f/8$, $f/11$, and $f/16$. Any one of these settings can be selected by adjusting the aperture, which changes the value of D . Increasing the setting from one f -number to the next higher value (for example, from $f/2.8$ to $f/4$) decreases the area of the aperture by a factor of 2. The lowest f -number setting on a camera lens corresponds to a wide-open aperture and the use of the maximum possible lens area.

Simple cameras usually have a fixed focal length and a fixed aperture size, with an f -number of about $f/11$. This high value for the f -number allows for a large depth of field, meaning that objects at a wide range of distances from the lens form reasonably sharp images on the CCD. In other words, the camera does not have to be focused.

Quick Quiz 6 A camera can be modeled as a simple converging lens that focuses an image on the CCD, acting as the screen. A camera is initially focused on a distant object. To focus the image of an object close to the camera, must the lens be (a) moved away from the CCD, (b) left where it is, or (c) moved toward the CCD?

EXAMPLE 11. Finding the Correct Exposure Time

The lens of a digital camera has a focal length of 55 mm and a speed (an f -number) of $f/1.8$. The correct exposure time for this speed under certain conditions is known to be $\frac{1}{500}$ s.

(A) Determine the diameter of the lens.

SOLUTION

The f -number for a lens relates its focal length to its diameter.

$$D = \frac{f}{f\text{-number}} = \frac{55 \text{ mm}}{1.8} = 31 \text{ mm}$$

(B) Calculate the correct exposure time if the f -number is changed to $f/4$ under the same lighting conditions.

SOLUTION

The total light energy hitting the CCD is proportional to the product of the intensity and the exposure time. If I is the light intensity reaching the CCD, the energy per unit area received by the CCD in a time interval t is proportional to $I \Delta t$. Comparing the two situations, we require that $I_1 \Delta t_1 = I_2 \Delta t_2$, where t_1 is the correct exposure time for $f/1.8$ and Δt_2 is the correct exposure time for $f/4$.

$$I_1 \Delta t_1 = I_2 \Delta t_2 \rightarrow \frac{\Delta t_1}{(f_1\text{-number})^2} = \frac{\Delta t_2}{(f_2\text{-number})^2}$$

$$\Delta t_2 = \left(\frac{f_2\text{-number}}{f_1\text{-number}} \right)^2 \Delta t_1 = \left(\frac{4}{1.8} \right)^2 \left(\frac{1}{500} \text{ s} \right) \approx \frac{1}{100} \text{ s}$$

As the aperture size is reduced, the exposure time must increase.

7. The Eye

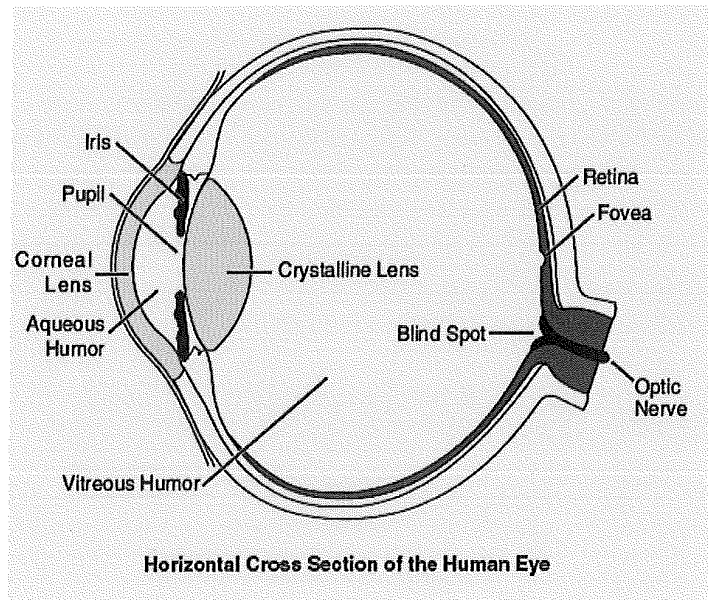


Figure 30 Important parts of the eye.

7.1. Anatomy of the Eye

The human eye achieves vision by forming an image that stimulates nerve endings, creating the sensation of sight. The eye consists of an aperture and lens system at the front, and a light-sensitive surface at the back. Light enters the eye through the aperture-lens system, and is focused on the back wall.

The lens system consists of two lenses:

- the **corneal lens** on the front surface of the eye, and
- the **crystalline lens** inside the eye.
- The space between the lenses is filled with a transparent fluid called the **aqueous humor**.
- Also between the lenses is the **iris**, an opaque, colored membrane.
- At the center of the iris is the **pupil**, a muscle-controlled, variable-diameter hole, or aperture, which controls the amount of light that enters the eye.

- The interior of the eye behind the crystalline lens is filled with a colorless, transparent material called the **vitreous humor**.
- On the back wall of the eye is the **retina**, a membrane containing light-sensitive nerve cells known as **rods** and **cones**. Rods are very sensitive to low light levels, but provide us only with low-resolution black-and-white vision. Cones allow us to see in color at higher resolution, but they require higher light levels.
- The **fovea**, a small area near the center of the retina, contains only cones and is responsible for the most acute vision.
- Surrounding the fovea is the sensitive region called the **macula**, which is responsible for central vision.
- Signals from the rods and cones are carried by nerve fibers to the **optic nerve**, which leads to the brain. The optic nerve connects to the back of the eye; there are no light-sensitive cells at the point where it attaches, resulting in a **blind spot**.

7.2. Accommodation

The eye focuses on an object by varying the shape of the pliable crystalline lens through a process called accommodation.

- The lens adjustments take place so swiftly that we are not even aware of the change.
- Accommodation is limited in that objects very close to the eye produce blurred images.
- The **near point** is the closest distance for which the lens can accommodate to focus light on the retina. This distance usually increases with age and has an average value of 25 cm. At age 10, the near point of the eye is typically approximately 18 cm. It

increases to approximately 25 cm at age 20, to 50 cm at age 40, and to 500 cm or greater at age 60.

- The **far point** of the eye represents the greatest distance for which the lens of the relaxed eye can focus light on the retina. A person with normal vision can see very distant objects and therefore has a far point that can be approximated as infinity.
- Only three types of color-sensitive cells are present in the retina. They are called red, green, and blue cones because of the peaks of the color ranges to which they respond (Fig.31). If the red and green cones are stimulated simultaneously (as would be the case if yellow light were shining on them), the brain interprets what is seen as yellow. If all three types of cones are stimulated by the separate colors red, blue, and green white light is seen. If all three types of cones are stimulated by light that contains all colors, such as sunlight, again white light is seen.

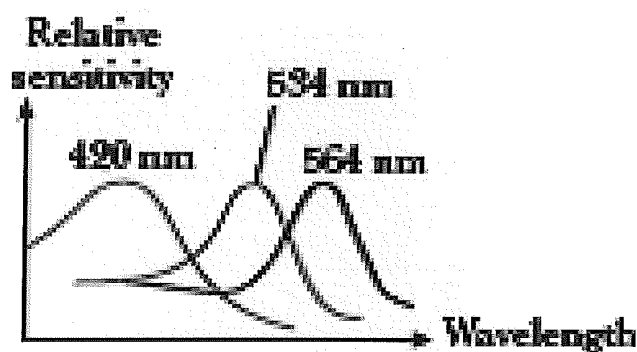


Figure 31 Approximate color sensitivity of the three types of cones in the retina

Color televisions take advantage of this visual illusion by having only red, green, and blue dots on the screen. With specific combinations of brightness in these three primary colors, our eyes can be made to see

any color in the rainbow. Therefore, the yellow lemon you see in a television commercial is not actually yellow, it is red and green! The paper on which this page is printed is made of tiny, matted, translucent fibers that scatter light in all directions, and the resultant mixture of colors appears white to the eye. Snow, clouds, and white hair are not actually white. In fact, there is no such thing as a white pigment. The appearance of these things is a consequence of the scattering of light containing all colors, which we interpret as white.

The power for the eye lens

The numerical value for the power of a lens in diopters is computed by taking the reciprocal of the focal length of a lens when the focal length is measured in meters. For example, a lens of 5-cm focal length has a power of 20 diopters.

- The human eye has a variable focal length so the eye can change its power to focus on objects at various distances from the eye.
- Let us calculate the accommodation ability of a typical human eye. If the distance from the eye lens to the retina is given by l (about 3 cm for a human eye), the power of the eye lens for **distant objects**, P_{∞} , can be calculated by using the thin lens formula

$$1/p + 1/q = 1/f$$

where p is the distance to the object, q is the distance to the image, and f is the focal length of the lens. For distant objects the reciprocal of the object distance $1/p$ is approximately zero, so the distance from the eye lens to the image on the retina is equal to the focal length of the eye. Therefore, the power for the eye lens for distant objects P_{∞} ,

$$P_{\infty} = 1/f = 1/l \cong 30 \text{ diopters}$$

where l is the distance from the eye lens to the retina.

- The power of the eye for **near objects**, P_n , can be similarly calculated,

$$P_n = 1/f = 1/p + 1/q$$

where p is the distance to an object at your near point (say, 25 cm) and q is the distance from the eye lens to the retinal,

$$P_n = 1/\text{near point} + 1/l = 1/0.25 + 1/l = 4 + P_{\infty} \cong 34 \text{ diopters}$$

Hence, the change in the power of a human eye lens to accommodate objects at various distances from the eye is about 4 diopters.

Conditions of the Eye

i- Farsightedness or Hyperopia

- When the eye suffers a mismatch between the focusing range of the lens–cornea system and the length of the eye, with the result that light rays from a near object reach the retina before they converge to form an image as shown in Figure 32a, the condition is known as farsightedness (or *hyperopia*).
- A farsighted person can usually see faraway objects clearly but not nearby objects. Although the near point of a normal eye is approximately 25 cm, the near point of a farsighted person is much farther away.
- The refracting power in the cornea and lens is insufficient to focus the light from all but distant objects satisfactorily.
- The condition can be corrected by placing a converging lens in front of the eye as shown in Figure 32b. The lens refracts the

incoming rays more toward the principal axis before entering the eye, allowing them to converge and focus on the retina. In other words, the lens forms a virtual image at the near point of the person.

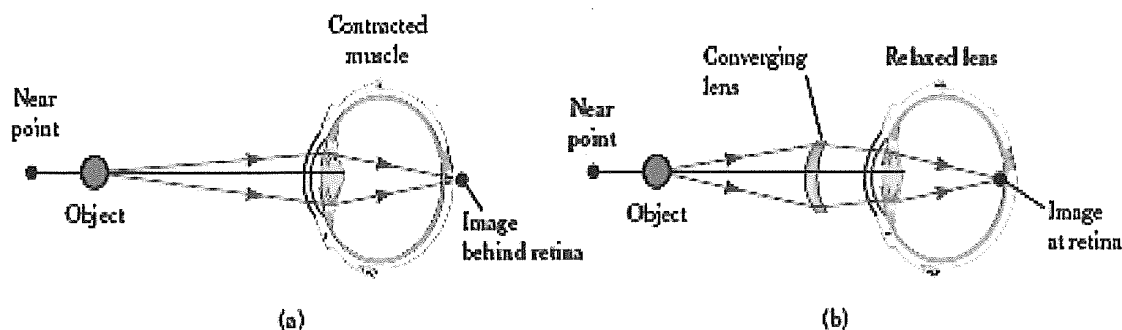


Figure 32 (a) When a farsighted eye looks at an object located between the near point and the eye, the image point is behind the retina, resulting in blurred vision. The eye muscle contracts to try to bring the object into focus. (b) Farsightedness is corrected with a converging lens.

EXAMPL : Correcting Hyperopia

Find the power of a lens designed for reading purposes to correct an eye with a near point of 75 cm. Assume a standard reading distance of 25 cm. Neglect the distance from the corrective lens to the cornea.

SOLUTION

A converging or positive lens is required, with a power such that when an object is placed 25 cm in front of the lens, the lens forms a virtual image of image distance $p = 75$ cm, so that this image serves as an object for the eye at the eye's near point. Applying the thin lens equation, we find

$$\frac{1}{f} = \frac{1}{p} - \frac{1}{q} = \frac{1}{0.25 \text{ m}} + \frac{1}{-0.75 \text{ m}} = 2.7 \Delta$$

ii- Nearsightedness or Myopia

- A person with nearsightedness (or myopia), another mismatch condition, can focus on nearby objects but not on faraway objects.
- The far point of the nearsighted eye is not infinity and may be less than 1 m . The maximum focal length of the nearsighted eye is insufficient to produce a sharp image on the retina, and rays from a distant object converge to a focus in front of the retina.
- They then continue past that point, diverging before they finally reach the retina and causing blurred vision (Fig. 33a).
- Nearsightedness can be corrected with a diverging lens as shown in Figure 33b. The lens refracts the rays away from the principal axis before they enter the eye, allowing them to focus on the retina.

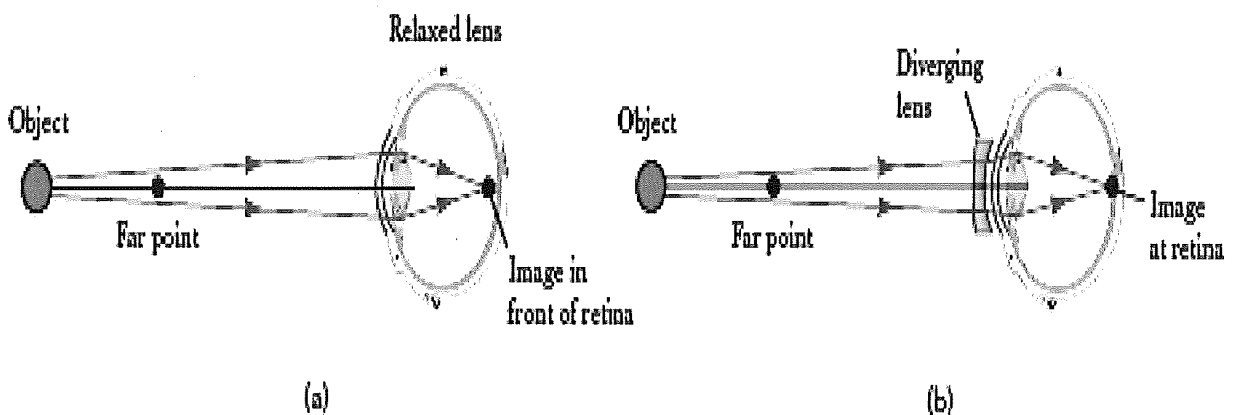


Figure 33 (a) When a nearsighted eye looks at an object that lies beyond the eye's far point, the image is formed in front of the retina, resulting in blurred vision. **(b)** Nearsightedness is corrected with a diverging lens.

EXAMPLE : Correcting Myopia

- (a) Find the optical power of a lens necessary to correct an eye with a far point of 50 cm. Neglect the distance from the lens to the cornea.
- (b) If the eye's near point is 10 cm, what is the eye's corrected near point?

SOLUTION

- (a) A diverging lens with a focal length of magnitude 50 cm is required.

$$f = 50 \text{ cm} = -0.50 \text{ m}$$

Thus the power of the lens is

$$P_o = \frac{1}{f} = \frac{1}{-0.5 \text{ m}} = -0.2 \Delta$$

- (b) An object will be at the "corrected near point" when the image produced by the lens is at the eye's actual near point.

Here the eye's near point is 10 cm in front of the lens. Thus we must find the object distance p for which the lens produces a virtual image 10 cm in front of the lens, that is, for which $q = 10 \text{ cm}$.

Applying the thin lens equation, we find

$$\begin{aligned}\frac{1}{p} &= \frac{1}{f} - \frac{1}{q} = P_o - \frac{1}{q} \\ &= -2.0 \Delta - \frac{1}{-0.10 \text{ m}} = +8.0 \text{ m}^{-1} \\ p &= \frac{1}{8.0 \text{ m}^{-1}} = 13 \text{ cm}\end{aligned}$$

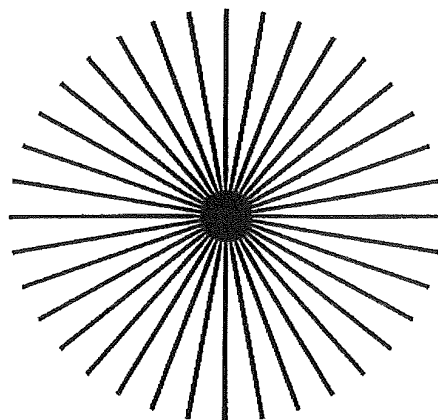
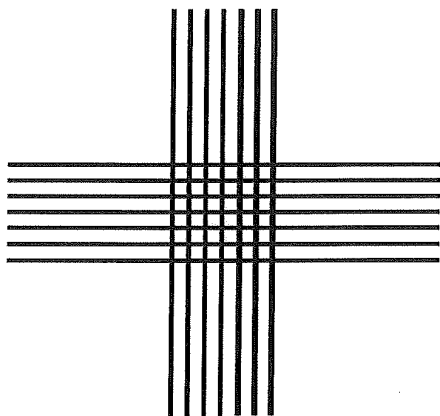
The original range of vision for this myopic eye was 10 cm to 50 cm. After correction, the range becomes 13 cm to ∞ .

iii- Presbyopia

- Beginning in middle age, most people lose some of their accommodation ability as their visual muscles weaken and the lens hardens. Presbyopia (literally, old age vision) is due to a reduction in accommodation ability.
- The cornea and lens do not have sufficient focusing power to bring nearby objects into focus on the retina.
- The symptoms are the same as those of farsightedness, and the condition can be corrected with converging lenses.

iv- Astigmatism

- Astigmatism is a very common refractive (focussing) error of the eyes which can cause blurred vision. It occurs as a result of irregular curvature on the surface of the cornea (corneal astigmatism) or in the lens of the eye itself.
- Astigmatism may cause blurring of both distant and near objects. You can be born with astigmatism or it may develop as you age. Astigmatism can also occur in combination with either near-sightedness (myopia) or far-sightedness (hyperopia).
- Astigmatism may be experienced as uneven blurring because the irregular curvature can make some parts of what you view more out of focus than other parts. By using the diagrams below, you can create an effect like that of astigmatism. If you are viewing the diagrams on a paper and when you tilt the top of the page away from you. Lines running across the page will appear less distinct than lines running up and down.



- The blur resulting from uncorrected astigmatism can make things uncomfortable to look at, difficult to focus on, and may cause headaches, tiredness and poor concentration. Your optometrist can prescribe glasses or contact lenses to correct your astigmatism and provide you with clear, comfortable vision.

Quick Quiz 7 Two campers wish to start a fire during the day. One camper is nearsighted, and one is farsighted. Whose glasses should be used to focus the Sun's rays onto some paper to start the fire? (a) either camper (b) the nearsighted camper (c) the farsighted camper

8. The Simple Magnifier

The simple magnifier, or magnifying glass, consists of a single converging lens. This device increases the apparent size of an object.

- Suppose an object is viewed at some distance p from the eye as illustrated in Fig.34.

- The size of the image formed at the retina depends on the angle θ subtended by the object at the eye. As the object moves closer to the eye, θ increases and a larger image is observed.
- An average normal human eye, however, cannot focus on an object closer than about 25 cm, the near point Fig.35a,
- Therefore, θ is maximum at the near point.

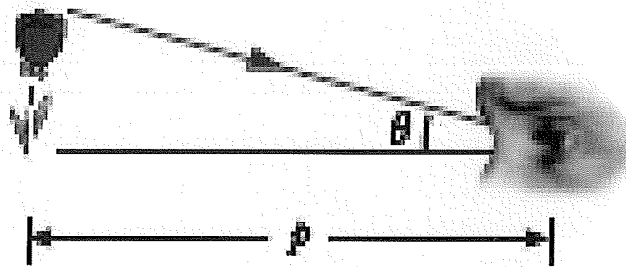


Figure 34 The size of the image formed on the retina depends on the angle θ subtended at the eye.

- To further increase the apparent angular size of an object, a converging lens can be placed in front of the eye as in Fig. 35b, with the object located at point O, immediately inside the focal point of the lens.
- At this location, the lens forms a virtual, upright, enlarged image. We define angular magnification m as the ratio of the angle subtended by an object with a lens in use (angle θ in Fig. 35b) to the angle subtended by the object placed at the near point with no lens in use (angle θ_0 in Fig. 35a):

$$m = \frac{\theta}{\theta_0} \quad (22)$$

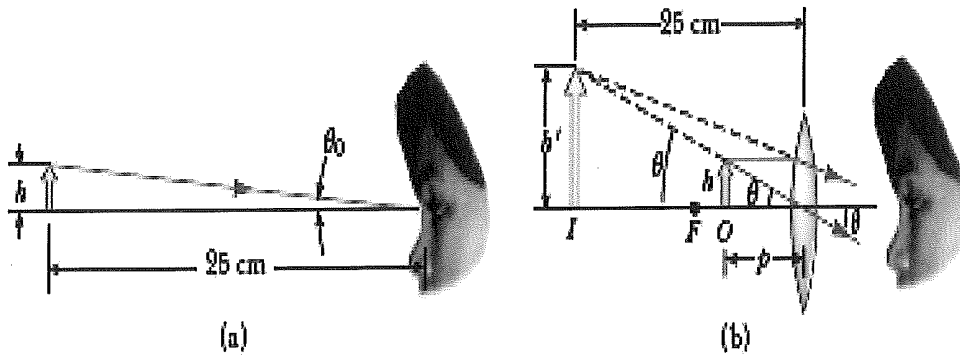


Figure 35 (a) An object placed at the near point of the eye ($p = 25 \text{ cm}$) subtends an angle $\theta_0 \approx h/25$ at the eye. (b) An object placed near the focal point of a converging lens produces a magnified image that subtends an angle $\theta \approx h'/25$ at the eye.

The angular magnification is a maximum when the image is at the near point of the eye, that is, when $q = -25 \text{ cm}$. The object distance corresponding to this image distance can be calculated from the thin lens equation:

$$\frac{1}{p} + \frac{1}{-25 \text{ cm}} = \frac{1}{f} \rightarrow p = \frac{25f}{25 + f}$$

where f is the focal length of the magnifier in centimeters. If we make the small angle approximations

$$\tan \theta_0 \approx \theta_0 \approx \frac{h}{25} \quad \text{and} \quad \tan \theta \approx \theta \approx \frac{h}{p} \quad (23)$$

Equ.22 becomes

$$m_{\max} = \frac{\theta}{\theta_0} = \frac{h/p}{h/25} = \frac{25}{p} = \frac{25}{25f/(25 + f)}$$

$$m_{\max} = 1 + \frac{25 \text{ cm}}{f} \quad (24)$$

Although the eye can focus on an image formed anywhere between the near point and infinity, it is most relaxed when the image is at infinity. For the image formed by the magnifying lens to appear at infinity, the object has to be at the focal point of the lens. In this case, Equations 23 become

$$\theta_o \approx \frac{h}{25} \quad \text{and} \quad \theta \approx \frac{h}{f}$$

and the magnification is

$$m_{min} = \frac{\theta}{\theta_o} = \frac{25 \text{ cm}}{f} \quad (25)$$

With a single lens, it is possible to obtain angular magnifications up to about 4 without serious aberrations. Magnifications up to about 20 can be achieved by using one or two additional lenses to correct for aberrations.

EXAMPLE 12. Magnification of a Lens

What is the maximum magnification that is possible with a lens having a focal length of 10 cm, and what is the magnification of this lens when the eye is relaxed?

SOLUTION

In Fig. 35b for the situation in which a magnifying glass forms an enlarged image of an object placed inside the focal point. The maximum magnification occurs when the image is located at the near point of the eye. When the eye is relaxed, the image is at infinity.

$$m_{max} = 1 + \frac{25 \text{ cm}}{f} = 1 + \frac{25 \text{ cm}}{10 \text{ cm}} = 3.5$$

$$m_{min} = \frac{25 \text{ cm}}{f} = \frac{25 \text{ cm}}{10 \text{ cm}} = 2.5$$

9. The Compound Microscope

A simple magnifier provides only limited assistance in inspecting minute details of an object.

- Greater magnification can be achieved by combining two lenses in a device called a compound microscope shown in Fig. 36a.
- It consists of one lens, the objective, that has a very short focal length $f_o < 1$ cm and a second lens, the eyepiece, that has a focal length f_e of a few centimeters. The two lenses are separated by a distance L that is much greater than either f_o or f_e .
- The object, which is placed just outside the focal point of the objective, forms a real, inverted image at I_1 , and this image is located at or close to the focal point of the eyepiece.
- The eyepiece, which serves as a simple magnifier, produces at I_2 a virtual, enlarged image of I_1 .
- The lateral magnification M_1 of the first image is $-q_1 / p_1$.

Notice from Active Fig. 36a that q_1 is approximately equal to L and that the object is very close to the focal point of the objective: $p_1 \approx f_o$. Therefore, the lateral magnification by the objective is

$$M_o \approx -\frac{L}{f_o}$$

- The angular magnification by the eyepiece for an object (corresponding to the image at I_1) placed at the focal point of the eyepiece is, from Equation 25,

$$m_e = \frac{25 \text{ cm}}{f_e}$$

- The overall magnification of the image formed by a compound microscope is defined as the product of the lateral and angular magnifications:

$$M = M_o m_e = -\frac{L}{f_o} \left(\frac{25 \text{ cm}}{f_e} \right)$$

The negative sign indicates that the image is inverted.

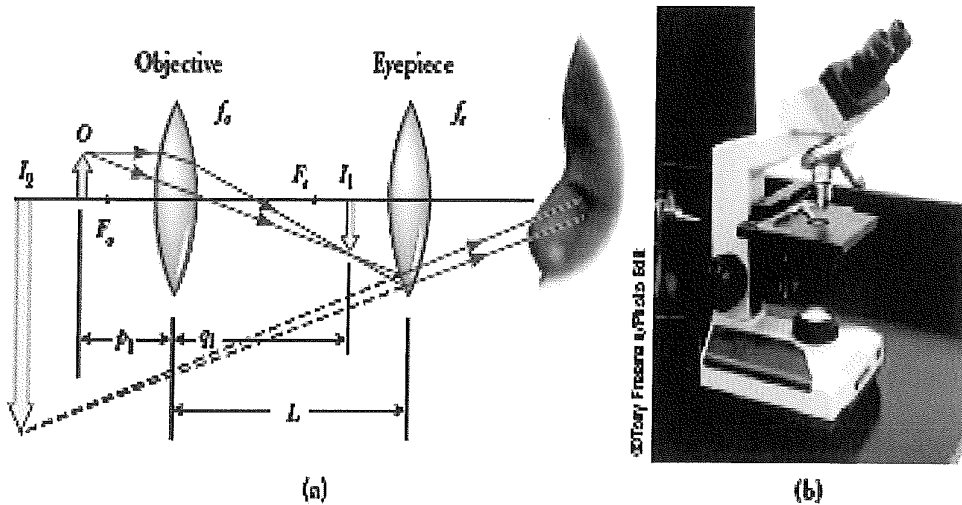


FIGURE 36. (a) Diagram of a compound microscope, which consists of an objective lens and an eyepiece lens. (b) A compound microscope.

The microscope has extended human vision to the point where we can view previously unknown details of incredibly small objects. The capabilities of this instrument have steadily increased with improved techniques for precision grinding of lenses. A question often asked about microscopes is, “If one were extremely patient and careful, would it be possible to construct a microscope that would enable the human eye to see an atom?” The answer is no, as long as light is used to illuminate the object. For an object under an optical microscope (one that uses visible light) to be seen, the object must be at least as large as a wavelength of light. Because the diameter of any atom is many times smaller than the wavelengths of visible light, the mysteries of the atom must be probed using other types of “microscopes.”

10. The Telescope

Two fundamentally different types of telescopes exist; both are designed to aid in viewing distant objects, such as the planets in our solar system. The refracting telescope uses a combination of lenses to form an image, and the reflecting telescope uses a curved mirror and a lens.

- Like the compound microscope, the refracting telescope shown in Active Figure 37a has an objective and an eyepiece.
- The two lenses are arranged so that the objective forms a real, inverted image of a distant object very near the focal point of the eyepiece.
- Because the object is essentially at infinity, this point at which I_1 forms is the focal point of the objective.
- The eyepiece then forms, at I_2 , an enlarged, inverted image of the image at I_1 .
- To provide the largest possible magnification, the image distance for the eyepiece is infinite. The light rays exit the eyepiece lens parallel to the principal axis, and the image due to the objective lens must form at the focal point of the eyepiece. Hence, the two lenses are separated by a distance $f_o + f_e$, which corresponds to the length of the telescope tube.
- The angular magnification of the telescope is given by θ/θ_o , where θ_o is the angle subtended by the object at the objective and θ is the angle subtended by the final image at the viewer's eye.
- Consider Active Figure 37a, in which the object is a very great distance to the left of the figure. The angle θ_o (to the left of the objective) subtended by the object at the objective is the same as

the angle (to the right of the objective) subtended by the first image at the objective. Therefore,

$$\tan \theta_o \approx \theta_o \approx -\frac{h'}{f_o}$$

where the negative sign indicates that the image is inverted.

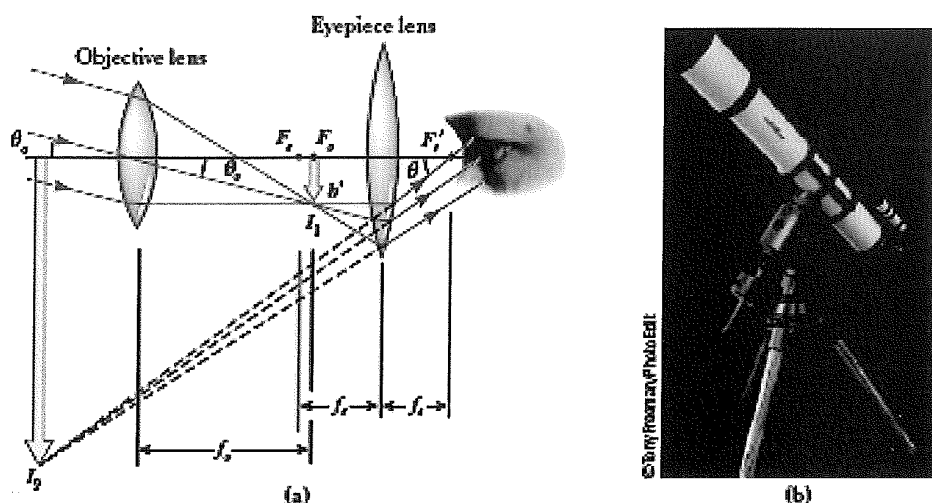


FIGURE 37. (a) Lens arrangement in a refracting telescope, with the object at infinity. (b) A refracting telescope.

The angle θ subtended by the final image at the eye is the same as the angle that a ray coming from the tip of I_1 and traveling parallel to the principal axis makes with the principal axis after it passes through the lens. Therefore,

$$\tan \theta \approx \theta \approx \frac{h'}{f_e}$$

We have not used a negative sign in this equation because the final image is not inverted; the object creating this final image I_2 is I_1 , and both it and I_2 point in the same direction. Therefore, the angular magnification of the telescope can be expressed as

$$m = \frac{\theta}{\theta_o} = \frac{h'/f_e}{h'/f_o} = -\frac{f_o}{f_e} \quad (27)$$

- This result shows that the angular magnification of a telescope equals the ratio of the objective focal length to the eyepiece focal length.
- The negative sign indicates that the image is inverted.
- When you look through a telescope at such relatively nearby objects as the Moon and the planets, magnification is important.
- Individual stars in our galaxy, however, are so far away that they always appear as small points of light no matter how great the magnification.
- To gather as much light as possible, large research telescopes used to study very distant objects must have a large diameter.
- It is difficult and expensive to manufacture large lenses for refracting telescopes.
- Another difficulty with large lenses is that their weight leads to sagging, which is an additional source of aberration.
- These problems associated with large lenses can be partially overcome by replacing the objective with a concave mirror, which results in a reflecting telescope.
- Because light is reflected from the mirror and does not pass through a lens, the mirror can have rigid supports on the back side. Such supports eliminate the problem of sagging.

References

- 1- *Physics for Scientists and Engineers with Modern Physics, Seventh Edition,*
Raymond A. Serway and John W. Jewett, Jr. Thomson Brooks/cole, 2008
- 2- *Physics Including Human Applications Chapter 20- Human Vision, page 440*
- 3- <http://www.astarmathsandphysics.com>
- 4- <http://www.cusd200.org>